Efficient integral equation based analysis of propagation in a simplified urban environment

*Conor Brennan, Peter J. Cullen and John Volakis**

Department of Electrical Engineering
Trinity College Dublin, Ireland
(*) Department of Electrical Engineering and Computer Science
University of Michigan, Ann Arbor, MI, USA
Tel. 353-1-6083810, Fax 353-1-6772442, E-mail brennajc@tcd.ie

Abstract— **We present a robust integral equation based iterative technique for the calculation of UHF radiowave field strengths within a simplified model urban environment composed of perfectly conducting planar screens. The solution algorithm uses a nested version of the forward/backward iterative technique. The fast far-field algorithm is introduced to calculate fields scattered between far-away screens. Numerical results are presented illustrating the application of the technique to a realistically sized problem.**

I. Introduction

The prediction of UHF electromagnetic wave propagation within a urban environment is of central importance to the planning of wireless communication systems. The uniform theory of diffraction (UTD) is the most commonly used technique for computing these fields. UTD is an asymptotic solution which postulates that energy propagates along direct, reflected and diffracted rays. In contrast integral equations can provide full-wave solutions, though the high computational overhead associated with them has meant that they have been applied primarily to physically small problems. However, recent research in two complementary areas has made integral equation analysis more tractable. The first area is the investigation of efficient iteration schemes for the solution of the associated matrix equations. Iterative techniques such as the forward/backward method [1], also known as the method of ordered multiple interactions (MOMI)[2] build the solution in an intuitively appealing manner which is reminiscent of the physical propagation of the waves. The second area has been acceleration algorithms which enable us to compute these iterative solutions more efficiently. Multipole methods such as the FAFFA[3] have had much success in the acceleration of iterative solutions. This paper applies efficient integral equation techniques to the modelling of UHF radiowave propagation through a simplified urban environment. The paper is arranged as follows. Section (II) describes the simplified urban model that we adopt and introduces the electric field integral equation and method of moments numerical solution. Section (III) describes the basic ideas behind the forward/backward iterative scheme. These ideas are extended to the urban propagation problem in section (IV). While the forward/backward technique is an efficient way of building the solution, it still has many computational bottlenecks, particularly the calculation of fields scattered between screens. We use the fast far-field algorithm to reduce this computational burden. The results of numerical experiments which illustrate the technique's application to large urban propagation problems are presented in section (V).

II. Microcell model and integral equation formulation

We model the buildings as perfectly conducting screens of infinite extent in the z-direction (i.e. our model is two-dimensional). The screens are laid out horizontally and vertically in the (x, y) plane in a grid-like structure as one would find in a typical urban area such as that depicted in Figure 1. While

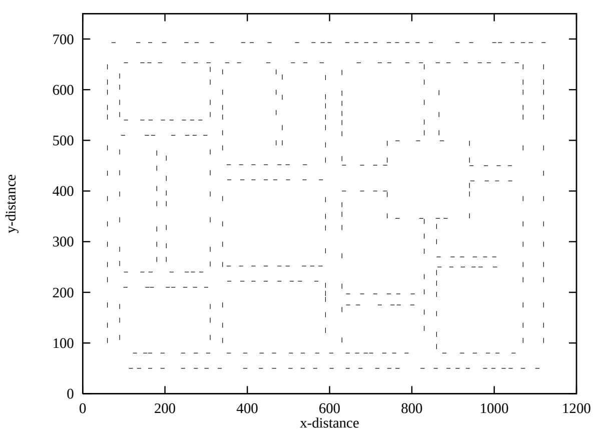

Fig. 1. Trial microcell layout used for numerical experiments in this paper. The source is located at (595, 460).

such an abstraction is obviously a dramatic simplification of the physical problem it is sufficiently rich in the key aspects of radiowave propagation that we wish to model, namely reflection from and diffraction around buildings. Future work will extend the ideas presented here to more realistic urban models. Our source is a TM^z line-source radiating time-harmonic fields with time-dependancy $e^{j\omega t}$. We use the two-dimensional electric field integral equation (EFIE) as our problem formulation.

$$E^i(\mathbf{r}) = \frac{\beta\eta}{4}\int_C J(\mathbf{r}')H_0^{(2)}\left(\beta\left|\mathbf{r}-\mathbf{r}'\right|\right)d\mathbf{r}' \qquad (1)$$

The integration contour C ranges over the screens. $\mathbf{r}$ and $\mathbf{r}'$ are, respectively, field points and integration points on the scatterer surface. $E^i(\mathbf{r})$ is the incident field at $\mathbf{r}$, defined to be the field at that point in the absence of any scatterers. η is the free-space impedance which has a value of 377Ω. Introducing N pulse basis functions and Dirac-delta testing functions discretises the problem into the matrix equation

$$ZJ = V \qquad (2)$$

where

$$\begin{aligned} Z_{ij} &= \frac{\beta\eta}{4}H_0^{(2)}\left(\beta\left|\mathbf{r}_i-\mathbf{r}_j\right|\right)\Delta_j & (3)\\ Z_{ii} &= \frac{\beta\eta\Delta_i}{4}\left(1-\jmath\frac{2}{\pi}\log\frac{1.781\beta\Delta_i}{4e}\right) & (4)\end{aligned}$$

V is a vector of size N containing the incident field at the centre of the basis domains. Z is an $N\times N$ matrix containing information about the interactions between the basis functions. The large number of basis functions typically used means that the order of the impedance matrix Z is very large for typical problems. This precludes direct inversion and equation (2) is generally solved using an iterative solver such as the conjugate gradient method. An intuitively appealing iterative solver which has provoked interest recently is the forward/backward method. This solution method partitions the impedance matrix into a sum of strictly upper and lower triangular matrices along with the diagonal matrix. These matrices are used to generate an infinite series solution for the surface current. High order terms in the series correspond physically to energy that has travelled forwards and backwards along the scatterer surface several times. Convergence is never rigorously achieved as that would require the summation of an infinite number of terms. However for most situations it is possible to achieve accurate results in only a few iterations. In the next section we will illustrate a slightly more general implementation of the forward/backward solution.

III. Block-based Forward backward solution of the matrix equation

Before applying a forward/backward iterative method to our urban propagation problem let us examine its workings in the general context of scattering by three scatterers, or collections of scatterers. The extension to an arbitrary number of scatterers is straightforward. We can define an impedance matrix, Z, for this three body problem along with the matrix equation as discussed in the previous section. Let $\tilde{Z}_{mn}$ refer to the sub-matrix of Z containing interactions between the basis functions residing on scatterer m and scatterer n. Similarly let $\tilde{V}_m$ be a subvector of V containing the incident field on scatterer m. $\tilde{J}_m$ is a subvector of J containing the amplitudes of the basis functions residing on scatterer m. The block forward/backward method proceeds by solving for the current on scatterer one in the absence of the other scatterers. This current, in addition to the source, is used to excite a field on scatterer two. This field excites a current on scatterer two which can be computed. The next step is to use the source and currents calculated thusfar to excite a field on scatterer three. This field excites a current which can be computed. This initial forward sweep corresponds to the sequential solution of the following equations

$$\begin{aligned} \tilde{Z}_{11}\tilde{J}_1 &= \tilde{V}_1 & (5)\\ \tilde{Z}_{22}\tilde{J}_2 &= \tilde{V}_2-\tilde{Z}_{21}\tilde{J}_1 & (6)\\ \tilde{Z}_{33}\tilde{J}_3 &= \tilde{V}_3-\tilde{Z}_{31}\tilde{J}_1-\tilde{Z}_{32}\tilde{J}_2 & (7)\end{aligned}$$

Upon solving for $\tilde{J}_3$ we allow backscatter to propagate through the system. This is achieved by sequentially solving the following equations.

$$\begin{aligned} \tilde{Z}_{22}\tilde{J}_2 &= \tilde{V}_2-\tilde{Z}_{21}\tilde{J}_1-\tilde{Z}_{23}\tilde{J}_3 & (8)\\ \tilde{Z}_{11}\tilde{J}_1 &= \tilde{V}_1-\tilde{Z}_{12}\tilde{J}_2-\tilde{Z}_{13}\tilde{J}_3 & (9)\end{aligned}$$

After having calculated the backscattered fields we again compute forward propagating fields by sequentially solving

$$\begin{aligned} \tilde{Z}_{22}\tilde{J}_2 &= \tilde{V}_2-\tilde{Z}_{21}\tilde{J}_1-\tilde{Z}_{23}\tilde{J}_3 & (10)\\ \tilde{Z}_{33}\tilde{J}_3 &= \tilde{V}_3-\tilde{Z}_{31}\tilde{J}_1-\tilde{Z}_{32}\tilde{J}_2 & (11)\end{aligned}$$

Note that in each equation, when recomputing the current on a given screen, we use the most up-to-date current estimates available for each of the other screens. This process of calculating forward and back propagating fields is continued for as many iterations as is deemed necessary. Note that each iteration involves updating the fields impinging on a scatterer and solving for the updated current. At each step we must solve a matrix equation for the current on scatterer m. Depending on the size of the $\tilde{Z}_{mm}$ submatrix this could be done by direct matrix inversion, using a conjugate gradient solver or even applying a forward/backward technique within scatterer m itself. Indeed, it is possible to formulate an algorithm based on a series of nested forward/backward solvers. Each forward/backward solver could solve for the fields propagating in a small subregion of the problem and the fields thus calculated can be marched from subregion to subregion. We discuss this nested application in more detail in the next section.

IV. Application of nested forward/backward iterative scheme to urban propagation problem

The forward/backward current marching scheme of the previous section could be readily applied to the urban propagation problem by treating each screen as a scatterer and proceeding as outlined. Instead we found it useful to apply a nested forward/backward algorithm. To do this we divide the area of interest into a grid structure as indicated in Figure 2. Each grid square contains several screens. At the highest level (level 4) the algorithm proceeds by applying the forward/backward algorithm to the rows of the grid. Specifically we identify the rows as the scatterers of section (III) and march the currents from row to row. Note that at each step of the iteration process we must solve for the currents in a row excited by the source and from other rows. This involves solving a matrix equation of the form

$$\tilde{Z}_{mm}\tilde{J}_m = \hat{\tilde{V}}_m \tag{12}$$

where

$$\hat{\tilde{V}}_m = \tilde{V}_m - \sum_{n \neq m} \tilde{Z}_{mn}\tilde{J}_n$$

$\hat{\tilde{V}}_m$ is the field on row m from the source and from other rows. The submatrix $\tilde{Z}_{mm}$ contains information about the interactions of all basis functions residing within row m and is of quite a large order. Solving matrix equation (12) is thus not a trivial task. However, just as we solve the entire problem by marching currents from row to row, we can solve each individual row problem by marching currents between grid-squares within the row as indicated by the level three portion of Figure 2. This idea of nesting forward/backward algorithms can be taken a step further by using a forward/backward scheme to calculate the currents on screens within a grid-square (level 2). We could apply yet another forward/backward algorithm to the level 1 problem of calculating currents on each individual screen. Instead we found it useful to use a conjugate gradient solver, or to exploit the planar nature of the screens and use a fast Fourier transform technique. By employing such a nested sequence of forward/backward algorithms we are able to ensure that we include many interactions between screens which are physically nearby (within the same grid-square) while including less interactions between screens that are physically far removed. While this nested algorithm orders the interactions in an efficient manner we still are faced with the computationally intensive task of numerically calculating them. To tackle this computational bottleneck we introduced the fast far-field algorithm (FAFFA). The pulse basis functions on each screen are divided into groups, each with a designated centre. In essence the FAFFA approximates the field propagating between groups on far-away screens as a plane wave. The amplitude of the plane wave is determined by numerically calculating the field scattered to the centre of the receiving group. The plane wave structure means that fields scattered to all other points in the receiving group are a phase-shifted version of the field scattered to the group centre and are thus easy to compute. This removes the need for their independent numerical computation. A detailed discussion of this algorithm is available in [3].

V. Numerical Results

The first numerical experiment conducted involved the test microcell depicted in Figure 1. There are 391 screens of length $10m$ in this problem. The line-source was located at (595,460) and radiated at 100MHz. The surface current was first calculated using a conjugate gradient solver. Total fields were calculated using this reference current along a test path which ran from (75,200) to (75, 235). The experiment was intended to illustrate the convergence properties of the nested forward/backward iterative method. A grid of $50m \times 50m$ squares was superimposed over the computational area. A conjugate gradient solver was used to calculate currents on each screen (level one problems). Two forward/backward iterations were used to solve level two problems, that is propagation between screens within each square. Two forward/backward iterations were used for level three problems, that is propagating fields from square to square within each row. The number of between-row forward/backward iterations (level 4 interactions) was varied to examine the convergence properties of the algorithm. Fields were calculated along the test line after 1 complete forward/backward row iteration, 3 complete forward/backward row iterations and 9 complete forward/backward row iterations. These fields are plotted along with the reference fields in Figure 3. The agreement with the reference fields is very good after 3 row iterations, and is almost perfect after 9. In order to test the technique on a more challenging problem we raised the frequency of the transmitting source to 970 MHz. At this frequency there are over 100,000 unknowns in the problem. The nested technique was used as described above with 5 forward/backward row iterations. In addition we used the FFT to calculate currents on each screen and used the FAFFA to calculate fields scattered between screens which are separated by more than 50m. Each screen was divided into 16 groups to implement the FAFFA. Fields were calculated along a test line running from (75,200) to (75,215) and are plotted in Figure 4. Also plotted are reference results obtained using a standard conjugate gradient solver.

VI. Conclusion

We have presented an integral equation based technique for modelling UHF radiowave propagation within an urban environment. A grid is superimposed over the problem geometry and the solution algorithm

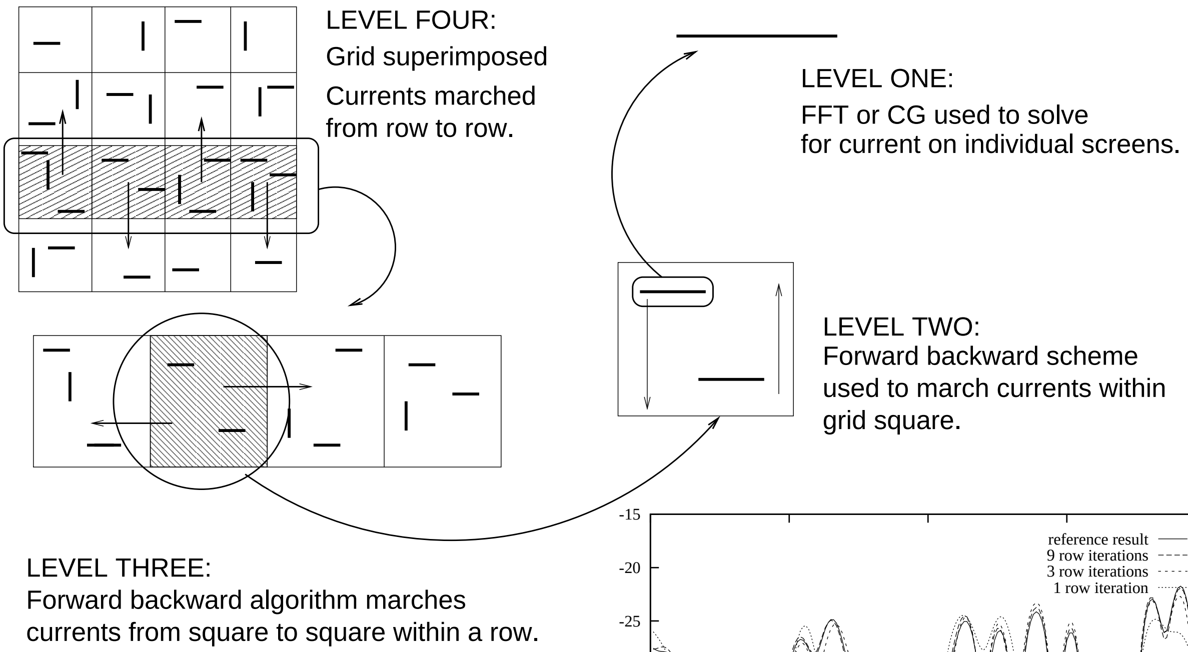

Fig. 2. Nested forward/backward algorithm. Grid superimposed over problem geometry. Forward/backward solution is applied to propagate fields between rows (level 4). Calculation of current within each row necessitates propagating fields between squares that constitute the row (level 3). This too is achieved using a forward/backward scheme. Fields propagated within each square can be calculated using a further nested forward/backward algorithm (level 2). Currents residing on indvidual screens can be calculated using an FFT or CG solver (level 1).

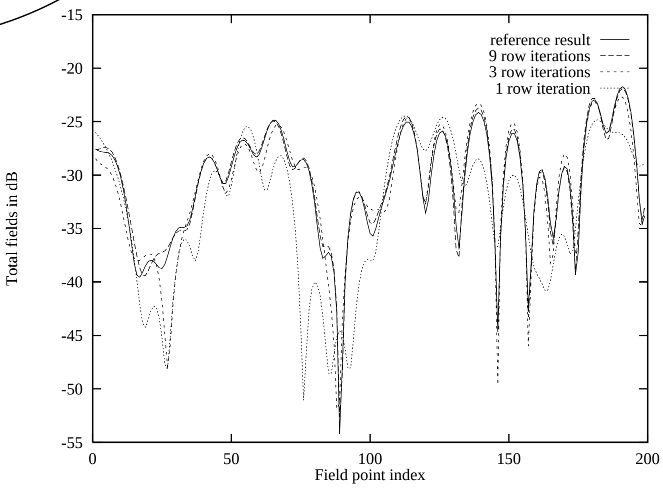

Fig. 3. Fields calculated along trial path form (75,200) to (75,235). Increased number of row to row iterations leads to incresed convergence of calculated fields.

proceeds by applying a nested version of the forward/backward iteration technique. The fast far-field algorithm was introduced to render field calculations more efficient. Numerical results were presented illustrating the accuracy of the technique when applied to a realistically sized problem. The ideas outlined can easily be extended to less abstracted models, such as propagation past a group of dielectric cylinders. It is hoped to extend the analysis to such a problem in the near future.

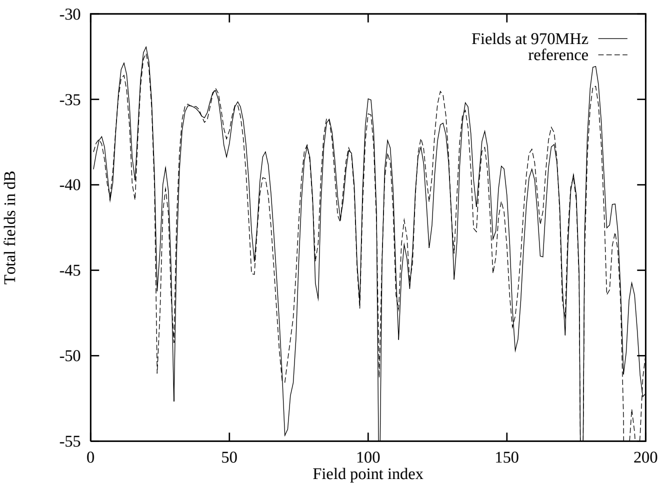

Fig. 4. Fields calculated at 970Mhz over trial path from (75,200) to (75,215).

VII. Acknowledgements

We acknowledge provision of funding by Enterprise Ireland through the Informatics research initiative. This initiative is supported by the National Development Plan 2000-2006.

References

[1] D. Holliday, l. L. De Raad Jr., G. J. St-Cyr,'Forward-backward: A new method for computing low-grazing angle scattering' IEEE Trans. Ant. Prop. Vol. 44 No. 5 pp. 722-729, 1996.
[2] D. Kapp and G. Brown,'A new numerical method for rough surface scattering calculations' IEEE Trans. Ant. Prop. Vol. 44 No. 5 pp. 711-722, 1996.
[3] C. C. Lu and W. C. Chew,'Fast Far-Field approximation or calculating the RCS of large objects' Microwave Opt. Tech. Lett. Vol. 8, no.5 pp.238-241 1995.