Validity Constraints and the TSP – GeneRepair of Genetic Algorithms

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Abstract

In this paper we present the outcome of a sets of experiments to evaluate the effectiveness of a new adjunct genetic operator GeneRepair. This operator was developed to correct invlaid tours which may be generated following crossover or mutation of our particular implementation of the genetic algorithm. Following implementation and testing of our genetic algotihm with GeneRepair we found a significant positive side in our results. Using GeneRepair along side traditional crossover and mutation operators we have been able to travers the search space of a problem and generate very good results in an extremely efficent manner, in both time and number of evaluations required.

KEY WORDS

Genetic Algorithms, TSP, Optimisation, Problem Constraints, GeneRepair.

1 Introduction

In this paper we present a novel approach to solving permutation problems that uses only standard crossover and standard mutation. We isolate the problem constraints in a separate operator, which operates as an adjunct operator to the standard set of genetic operators.
This approach is applicable to any problem domain where the solution constraints can be identified in the gene string. In this paper we explore two different types of permutation problems. We look at the Travelling Salesman Problem (TSP), which is a well-known NP-Complete problem [1]. The TSP involves visiting all cities on a map, generating the shortest total tour distance. As Mitchell [2] points out:

"*some type of encoding require specially defined crossover and mutation operators... like the Traveling Salesman Problem in which the task is to find a correct ordering for a collection of objects*".

2 Representation and Operators

The natural choice of representation for the TSP is an Order-based representation. These have been successfully applied to the TSP and the very similar Vechicle Routing Problem *VRP* problems by Fogel [3, 4, 5], Banzhaf [6], Ambati [7] and Pereira *et al* [8]. Additionally, the genetic operators employed must also be Order-based. If either the representation or the operators do not respect the Order based nature of the problem, then invalid solutions will be generated.

First, we looked at the crossover operators that respect the Order-based nature of permutation problems, and prevent the introduction of errors such as invalid tours [2]. The order preserving crossover operators that have been developed include: Order Crossover [9], Modified Crossover [10], Partially Mapped Crossover [11], Cycle Crossover [12], 2-quick / 2-repair [13], plus a number of less frequently used crossover operators [14].

Secondly, we looked at Order-based mutation operators developed for Order-based problems. These include: Displacement Mutation [15], Exchange Mutation [6], Insertion Mutation [3, 15], Simple Inversion Mutation [16, 17], Inversion Mutation [4, 5] and other order preserving mutation [18].

We present a solution for Order-based problems that uses only standard crossover and standard mutation. To counteract the invalid tours that occur as a result, we introduce GeneRepair - a genetic repair operator that has a number of positive effects: It allows the use of standard GA libraries, with the addition of a single repair operator for permutation problems. It simplifies the understanding of the GA, by allowing the use of standard crossover and mutation for Order-based problems. Finally, it removes problem specific activities from the genetic operators themselves, and isolates it in a single intra-generation operation.

3 GeneRepair

The GeneRepair enhanced genetic algorithm operates in the manner of traditional genetic algorithms, and can be summarized as follows:

1. Generate the initial population P(0) at random and set i = 0;
2. Evaluate the fitness of each individual in P(i);
3. Select parents from P(i) based on their fitness.
4. Apply standard crossover
5. Apply standard mutation.
6. Apply GeneRepair.
7. Repeat until convergence.

The TSP is NP-Complete, and may be characterized by two separate facets: Optimisation and Permutation. Responsibility for optimisation lies with the standard genetic algorithm, which effectively remains unchanged from Holland [16]. Responsibility for only allowing valid permutation in the population lies solely with the GeneRepair operator.

3.1 Solution Constraints

Combinatorial problems like the TSP place constrains on the valid solutions. Solutions are only valid when all N cities in the problem are present in the solution. Thus, we use a fixed-length chromosome to represent our tours. Furthermore, a solution is only considered valid when all cities are represented once only in the solution, and no cities are absent. These constraints act as a trigger for the application of the GeneRepair operator.
Non order-preserving crossover (above) can cause a violation of the validity constraint, by combining parent strings, which result in invalid offspring. See Figure 1.
Similarly, non order-preserving mutation operators can also generate invalid solutions. This happens when mutation randomly inserts a city that already exists in the solution.

Parent 1	0 1 2 3 4 | 5 6 7 8 | 9
Parent 2	8 4 1 6 3 | 7 9 2 0 | 5
Child 1	0 1 2 3 4 | 7 9 2 0 | 9
Child 2	8 4 1 6 3 | 5 6 7 8 | 5

Figure 1: Constraint violation by 2-Point Crossover.

In practice, GeneRepair examines each tour in turn, enforcing the following:

1. Correct number of cities in the tour
2. No duplicate cities
3. No missing cities

These constrains invoke the GeneRepair operator, and identifies the string the location of duplicate cities (see Figure 2).

Detection of invalid cities:
Child 1 0 1 2 3 4 7 9 2 0 9

Figure 2: GeneRepair- Invalid cities identified.

3.2 Repair

Knowing the location of the offending cities, GeneRepair replaces these cities iteratively with valid cities retrieved from a corrective template. The first strategy investigated was to replace the duplicate cities with the missing cities, according to a pre-determined template (see Figure 3).

GeneRepair Template	0 1 2 3 4 5 6 7 8 9
(i) Child 1	0 1 2 3 4 7 9 2 0 9
(ii)Child $1_{GeneRepaired}$	0 1 2 3 4 7 9 5 6 8

Figure 3: GeneRepair- correction of tour.

The majority of GeneRepair replacements were performed in a left-to-right manner - replacing the left-most duplicate city first. Additionally, the replacement city was retrieved from the template also in a left-to-right manner. However, brief evaluation of a random replacement technique, randomly selecting the replacement city from the template was also evaluated. Initial results show no identifiable difference between the two techniques.

The replaced city is selected according to a corrective template. Three different types of template were investigated:

1. Static template. This consisted of a preset valid tour, and remained constant throughout.
2. Parent-based Template. Select the fitter parent, and use that as the corrective template. This template varied for every corrected individual.

3. Random Template. For each corrected individual a new template of random numbers was generated, within the validity constraints of the TSP problem.

Each of these techniques was tested on a select number of TSP problems. The parent-based solution produced the worst results. Both random and fixed template solutions produced good results, with the randomly generated template producing marginally better results.

4 Experiment 1 - TSP

We evaluated GeneRepair on the TSP benchmark problems from the Heidelberg TSPLIB problem set [19]. We previously investigated the use of GeneRepair versus non-GeneRepair Genetic Algorithm TSP and VRP search [20]. For these experiments we investigated the potential of the GeneRepair based solution, without reference to a non-GeneRepair implementation.

We optimised the genetic parameters of crossover and mutation in order to produce the best solutions on selected TSP problems. This investigated the ability of GeneRepair to generate *optimal solutions only*, as the benchmark solutions are assured optimal solutions.

We conducted approximately 6 experiments on each of the 3 following problem sets, eli51, st70 and eil101. Throughout all experiments the population size was the square of the number of cities in the problem set. Tournament and roulette wheel selection (but not truncation selection) were used. Only 1-point and 2-point crossover was investigated. Exchange mutation was used exclusively, with rates varying between 0% and 10%.

The first problem set involved a 51-city TSP problem. Tests revealed the optimal mutation rate to be 0.75%.

The second problem involved 70 cities and optimality was found with mutation at 0.76%.

The final problem involved 101 cities and again a mutation rate of 0.76 was found to be the best. Figure 4 illustrates the test conducted for each of the problem set to obtain the optima mutation rate. An experiment was conducted to evaluate the effects of an adaptive mutation rate Figure 5 illustrates the effect of this adaptive mutation rate. The Genetic Algorithm clearly improves following a mutation rate change with improvements being made at approximately 3,500, 7,000 and 17,000 generations. The adaptive mutation rate initially changes at 5000 generations causing some diversity in the population and resulting in improvements in the tour length.

The number of repairs for the first thousand generations of the algorithm is initially high, even when a low mutation rate is applied. The percentage of repairs declines rapidly to between 1% and 2%, as can been seen in figure 6. It is important to note that with a low mutation rate, information in the population is exploited and the mutation effect of GeneRepair is embraced. GeneRepair in essence can be viewed as a second mutation operator whilst maintaining problem validity.

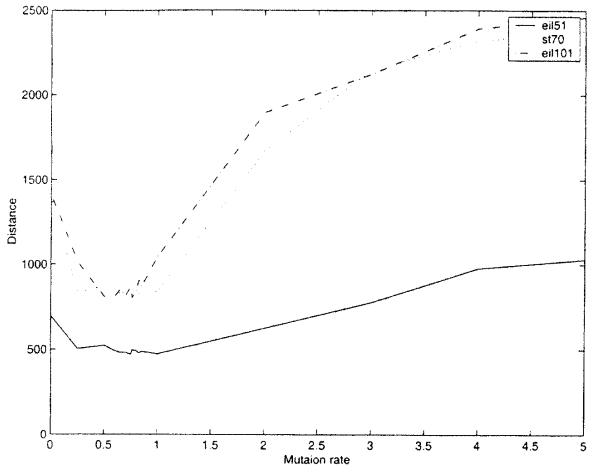

Figure 4: Distance at various mutation rates

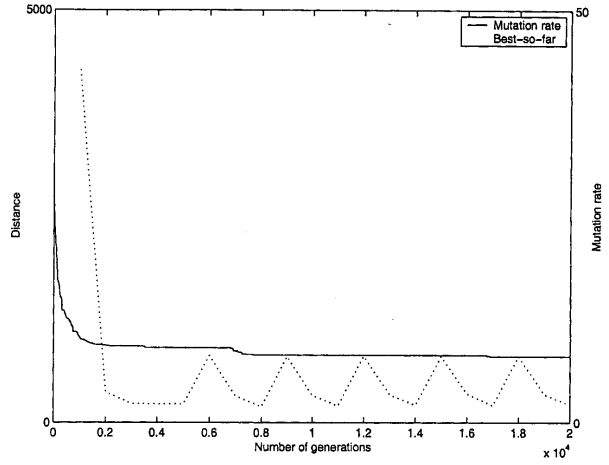

Figure 5: Adaptive mutation effect on GA convergence

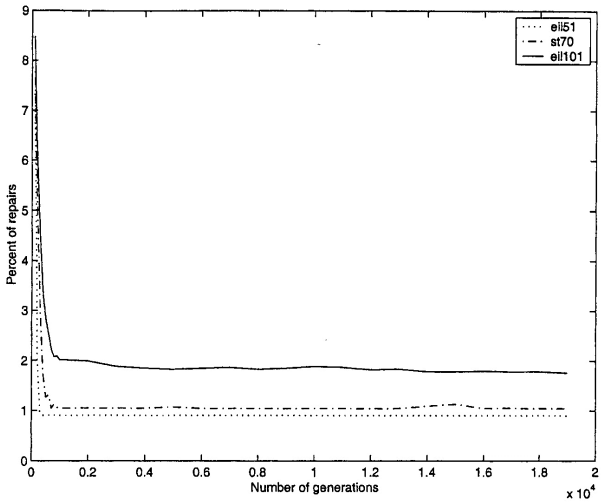

Figure 6: Percentage of repairs for each problem

Another important metric to be considered when examining GeneRepair enhanced Genetic Algorithms is the number of improvements versus mutation rate, this is illustrated in figure 7.

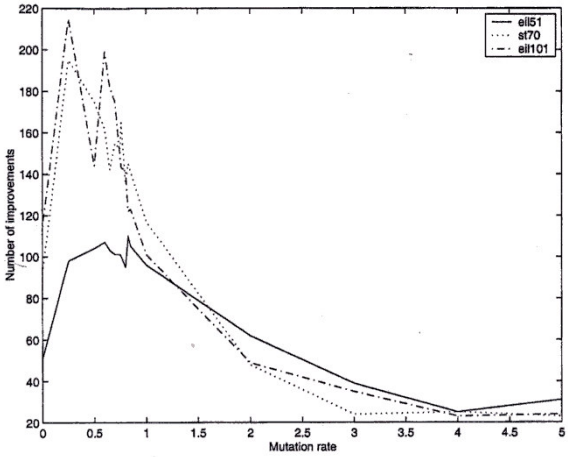

Figure 7: N^o of improvements at specific Mutation rates

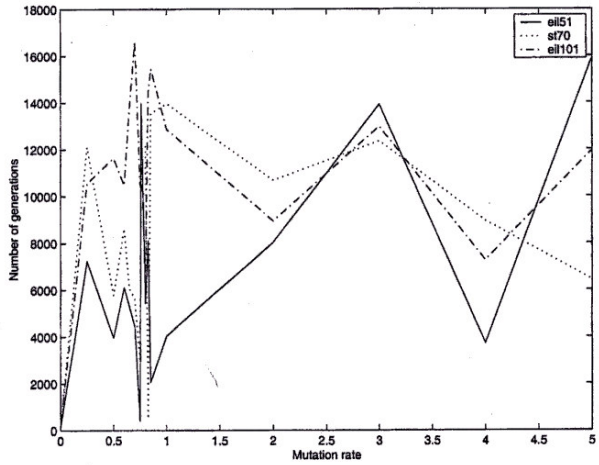

Figure 8: N^o of generations to find best average solution

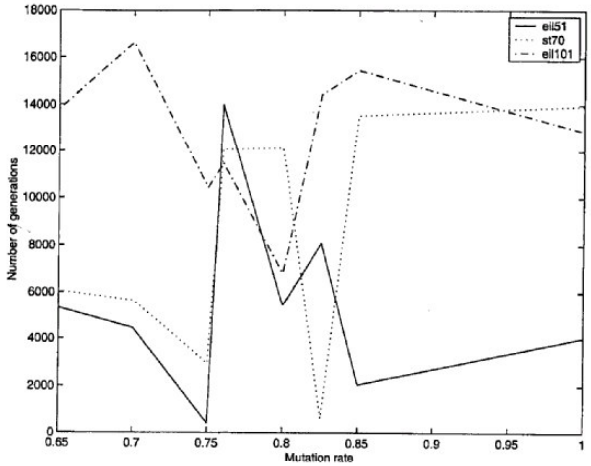

Figure 9: N^o of generations to find best average solution

Figure 7 and 8 illustrate how poor selection of mutation rate can lead to wasteful use of computation time. Figure 10 illustrates the impact that selection operator and crossover type together with mutation rate has on the best solution found.

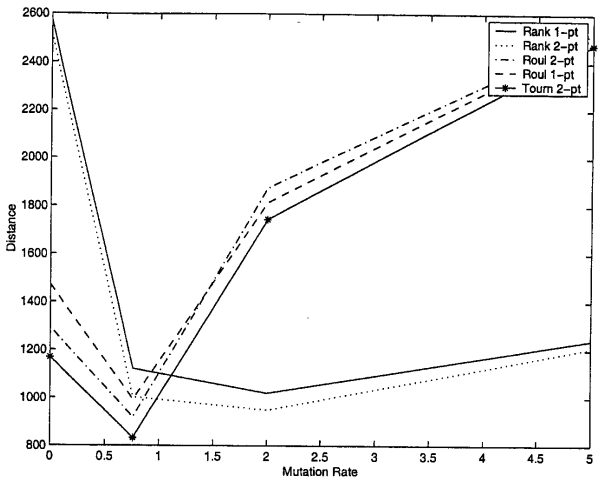

Figure 10: Effect of mutation, crossover & selection

5 Explanation for GENEREPAIR

GeneRepair is composed of two distinct tasks: *fault detection* and *fault correction*. To help identify the exact reason for GeneRepairs' improvement in performance, we analysed each phases in turn.

First we measure the frequency with which GeneRepair was invoked. GeneRepair repaired approximately 11% of the alleles, while solving the benchmark TSP problems. Additionally, some of these alleles required multiple repair operations. (As may be expected, these figures are higher during the first 100 epochs). For comparison, we recorded the number of invalid tours generated by our solution without GeneRepair [20]. Approximately 15% of individuals were found to violate the TSP validity constraint.

In general, GeneRepair does increase the number of generated individuals that form part of the valid search space. However, this relatively modest increase in the search space does not adequately account for the significant increase in performance obtained. For example, increasing the population size to allow for this 11% wastage, had little effect on the quality of the results generated.

Next we investigated the *fault correction* part of GeneRepair. First, we analyse how errors are introduced. Crossover introduces the majority of errors as it is always applied. It does this by combining incompatible sections of tours. (See figure 1)

N-point-Crossover preserves the identicallity between both parents. Thus, the GeneRepair operator is invoked

more during early evolution than it is when we reach convergence.
Secondly, the replacement strategy replaces invalid (i.e. duplicate) genes with missing genes, according to the replacement strategy described above. So, in conclusion, GeneRepair is a multi-point mutation operator, that is applied heavily during early evolution and rarely applied when convergence is achieved.
1-point mutation tends to introduce errors and, GeneRepair will Fix the error, but it does So randomly. Either the mutation will remain unaffected by GeneRepair and another duplicate city will be replaced. This has tie effect of causing 2-point mutation. Alternatively, the mutation itself will be repaired, which Reduces the level of mutation. Importantly, the mutation introduced by GeneRepair is Not an alternative to standard mutation, as standard mutation is still required when near-optimal convergence is reached. Initial results seem to indicate that the reduction in mutation is (at least partly) counteracted by GeneRepair's introduction of its own mutogenic effect, but investigations are ongoing.
This may account for our improved performance as it effectively prohibits the problem of premature convergence. Furthermore, it is applied less frequently during final convergence, allowing an optimal to be achieved. (This seems to mimic the operation of a Boltzman machine on simulated annealing problems.) However, investigations are at a relatively early phase, and research is ongoing.

6 Future work

The experiments performed so far highlight the need for a number of further investigations. Future work is necessary to compare the effectiveness of GeneRepair against the order-preserving crossover and mutation operators. We will also conduct experiments to evaluate the effectiveness of GeneRepair on large problems with more than 1000 cities. Finally, we will explore the interplay between standard mutation and the mutogenic effects of GeneRepair. This may involve the use of an adaptive mutation rate in conjunction with GeneRepair.

7 Conclusion

We solved a permutation problem by combining standard genetic operators with a novel genetic repair operator - GeneRepair. Validity constraints that originate in the problem domain are thus centralized in a single repair operator. We explored the use of GeneRepair on the TSP, using the fitness function to optimise the solution while GeneRepair ensures the validity of solutions. This approach is potentially applicable to any domain where the solution constraints can be separated from the fitness function. Results produced so far have either reached global optimal solutions, or have been close to optimal solutions. Furthermore, solutions appear to be produced in a relatively small number of generations. We examined the higher levels of early mutation that result from GeneRepair operations, as one possible explanation for the results produced so far.

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