Tabulated Interaction Method for Electromagnetic Wave Propagation in Rural Areas

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***Abstract*—In this paper an efficient method is proposed for computing electromagnetic wave propagation over terrain profiles. The paper extends the Tabulated Interaction Method (TIM) [1], applying it to the problem of 2D wave scattering from lossy surfaces using a coupled surface integral equation formulation. The proposed method is compared with the recently proposed Characteristic Basis Function Method (CBFM) [2], [3] with which it shares several features as well as the Forward Backward Method (FBM) which produces a numerically exact solution. Comparisons are made in terms of accuracy and run-time. Two types of terrain profiles, hilly and mountainous, were used to illustrate the robustness of the proposed method. The numerical analysis demonstrates that the TIM has a dramatically lower computational complexity and storage than the FBM and the CBFM while offering similar accuracy.**

***Index Terms*—Tabulated Interaction Method (TIM), Iterative method, Electromagnetic Scattering, Method of Moment (MoM), Integral Equation (IE), basis functions.**

I. INTRODUCTION

Efficient and accurate computation of electrogmanetic wave propagation remains a core requirement in wireless system planning. A wide range of propagation models have been proposed with integral equations (IE) offering particularly high accuracy. Typically, the relevant integral equation is discretised using the method of moments (MoM) resulting in a system of dense linear equations. The efficient numerical solution of these equations is a key topic in computational electromagnetics. Different techniques have been developed including iterative methods: Forward Backward Method (FBM) [4], Forward Backward Method with Spectral Acceleration (FBM-SA) [5], [6], acceleration methods such as the Fast Far Field Appoximation (FAFFA) [7], and compression techniques such as the Characteristic Basis Function Method (CBFM) [2], [3], etc. The Tabulated Interaction Method (TIM) is related to both the FAFFA and the CBFM and in this work we extend it, considering the propagation over lossy irregular terrain profiles.

II. FORMULATION

Consider an incident plane wave $\psi_{inc}(x,z)$ impinging upon a dielectric random surface with height profile $z=f(x)$. The upper medium is assumed to be free-space with permittivity ϵ_0 and wavenumber k_0 while the lower medium is assumed to be dielectric with permittivity ϵ_1 and wavenumber k_1. Assuming a time-variation of $e^{j\omega t}$ the electric fields in both regions can be described in terms of surface integral potentials [8]. Applying the appropriate boundary condition yields an integral equation in terms of the unknown fields on the scatterer surface. The MoM is applied using N pulse basis functions for the surface electric currents and point-matching, giving a discrete system of equations. It is assumed that the n^{th} pulse basis function is defined on a domain of length $\triangle_n$ with centre $\overrightarrow{r}_n$ and normal $\widehat{n}_n$. In the case of lossy dielectric surfaces, a system of coupled integral equations is applied. For a normal incident TM^z wave, the field components present are E_z, H_x and H_y. The pulse basis functions are used to represent the unknown electric current j_z and magnetic current k_t. Then, the MoM is applied, yielding a $2N\times 2N$ linear system

$$\begin{pmatrix} \overline{\overline{Z}}_a & \overline{\overline{Z}}_b \\ \overline{\overline{Z}}_c & \overline{\overline{Z}}_d \end{pmatrix}\begin{pmatrix} \overline{j} \\ \overline{k} \end{pmatrix} = \begin{pmatrix} \overline{e}_{inc} \\ 0 \end{pmatrix} \tag{1}$$

For completeness we provide the matrix non-diagonal entries below

$$Z_{mn}^{(a)} = \frac{k_0\eta_0}{4}H_0^{(2)}(k_0 r_{mn})\triangle_n \tag{2}$$

$$Z_{mn}^{(b)} = \frac{k_0}{4j}\hat{n}_n\cdot\hat{r}_{nm}H_1^{(2)}(k_0 r_{mn})\triangle_n \tag{3}$$

$$Z_{mn}^{(c)} = \frac{k_1\eta_1}{4}H_0^{(2)}(k_0 r_{mn})\triangle_n \tag{4}$$

$$Z_{mn}^{(d)} = \frac{k_1}{4j}\hat{n}_n\cdot\hat{r}_{nm}H_1^{(2)}(k_1 r_{mn})\triangle_n \tag{5}$$

where η_α is the impedance in medium α, $\gamma = 1.781$ and $r_{ij} = \sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$ Once found the surface electric currents can be used to compute scattered fields, and thus total fields, above the surface.

III. TABULATED INTERACTION METHOD

The TIM proceeds by partitioning the terrain profile into groups, each group having a designated group center. In the case of propagation over a terrain profile, the groups can be considered to be identical linear segments of fixed length. We also make the assumption of forward scattering. This is reasonable at grazing incidence and simplifies the discussion but is not strictly necessary as the TIM can also be formulated assuming back scattering. The forward scattering problem can be expressed as

$$\overline{\overline{Z}}_{ii}\overline{x}_i \quad = \quad \overline{b}_i - \overline{l}_i \quad \text{for}\, i=1...M \tag{6}$$

where M is the number of groups, $\overline{l}_i$ is a $\frac{2N}{M}\times 1$ vector containing the fields scattered in the forward direction to group

G_i. Specifically,

$$\bar{l}_i = \sum_{j<i} \overline{\overline{Z}}_{ij} \bar{x}_j \tag{7}$$

The matrix terms $\overline{\overline{Z}}_{ij}$ relate to the sub-matrix of $\overline{\overline{Z}}$ corresponding to interactions between group G_i and G_j. We note that the matrices on the diagonal $\overline{\overline{Z}}_{ii},\ i = 1...M$ are identical because the groups are locally flat and the same length. We use the notation $\overline{\overline{Z}}_{self}$ for these matrices. $\bar{x}_i$ and $\bar{b}_i$ are subvectors of $\begin{bmatrix} \bar{j} \\ \bar{k} \end{bmatrix}$ and $\begin{bmatrix} \bar{e}_{inc} \\ \bar{0} \end{bmatrix}$ respectively containing local current and local incident field information for group G_i. The proposed method expresses the right hand side (RHS) of equation (6) entirely as a sum of incoming plane waves, that is

$$\bar{b}_i - \bar{l}_i \equiv \sum_{k=1}^{K} \alpha_k^i \bar{p}^{(k)}$$

Having done this then $\bar{x}_i$ can be expressed as a weighted sum of basis functions

$$\bar{x}_i = \sum_{k=1}^{K} \psi_k^i \bar{q}^{(k)} \tag{8}$$

where the basis functions $\bar{q}^{(k)}$ are the currents induced on a typical linear segment excited by plane wave $\bar{p}^{(k)}$. These can be pre-computed and stored by solving the equations:

$$\overline{\overline{Z}}_{self} \bar{q}^{(k)} = \bar{p}^{(k)},\ \ k = 0...K \tag{9}$$

Note that the basis functions as defined are applicable to *all* groups. This is in contrast to the CBFM which creates individual basis functions for each individual group. We can recognise that the RHS of equation (6) contains two components, incidence field to group $\bar{b}_{g,i},\ i = 1...M$ and scattered field from previous groups to current group $\sum_{j<i} \overline{\overline{Z}}_{ij} \bar{x}_j$. In the next section we express these components in the form of a sum of plane waves as required.

A. Incidence field

Consider the unknown m, in group G_i with centre $\overrightarrow{r}_l$. We consider the far-field form of the Hankel function

$$H_\alpha^{(2)}(x) = \sqrt{\frac{2}{\pi x}} e^{-j(x - \alpha\frac{\pi}{2} - \frac{\pi}{4})} \tag{10}$$

For $r_{mn} \gg r_{lm}$, we can write

$$\begin{aligned} r_{mn} &\simeq r_{ln} - \hat{r}_{ln} \cdot \overrightarrow{r}_{lm} && (11)\\ &\simeq r_{ln} - \hat{r}_{ll'} \cdot \overrightarrow{r}_{lm} && (12)\\ &\simeq r_{ll'} - \hat{r}_{ll'} \cdot \overrightarrow{r}_{lm} - \hat{r}_{ll'} \cdot \overrightarrow{r}_{l'n} && (13)\\ &\simeq r_{ll'} - \hat{r}_{ll'} \cdot \overrightarrow{t}_m \Delta x - \hat{r}_{ll'} \cdot \overrightarrow{t}_n \Delta x && (14) \end{aligned}$$

where $\overrightarrow{r}_l$ and $\overrightarrow{r}_{l'}$ are the centres of group G_i and G_j. Inserting the approximation from equation (12) into the expressions of $\bar{b}_i$ and assuming the far field form of the Hankel function, allow us to write the incidence field from a source located at $\overrightarrow{r}_a$ to group G_l with center $\overrightarrow{r}_l$ as

$$\begin{aligned} \bar{b}_i &= \begin{bmatrix} \bar{e}_i^{inc} \\ \bar{0} \end{bmatrix} && (15)\\ &\simeq \frac{k_0\eta_0}{4}\sqrt{\frac{2}{\pi k_0 r_{al}}} e^{-j(k_0 r_{al} - \frac{\pi}{4})} \begin{bmatrix} \bar{p}_{q,0} \\ \bar{0} \end{bmatrix} && (16) \end{aligned}$$

where:

$$\bar{p}_{q,0} = \begin{bmatrix} e^{-k_0 \hat{r}_{la}\cdot\hat{t}_n \frac{Q}{2}\Delta x} \\ e^{-k_0 \hat{r}_{la}\cdot\hat{t}_n \frac{Q-1}{2}\Delta x} \\ . \\ . \\ e^{k_0 \hat{r}_{la}\cdot\hat{t}_n \frac{Q}{2}\Delta x} \end{bmatrix} \tag{17}$$

Equation (16) shows that the field incident on a group can be written in the form of plane wave and it is a function of distance from source to center of group and angle of incidence wave to group, $\hat{r}_{la} \cdot \hat{t}_i$. This plane wave can be expressed as an interpolation of two neighbouring pre-defined plane waves

$$\bar{b}_i \simeq \frac{k_0\eta_0}{4}\sqrt{\frac{2}{\pi k_0 r_{al}}} e^{-j(k_0 r_{al} - \frac{\pi}{4})} \sum_{k=0}^{K} \begin{bmatrix} \alpha^{(k)} \bar{p}_{q,0}^{(k)} \\ \bar{0} \end{bmatrix} \tag{18}$$

The right hand side of equation (18) has thus been expressed as a sum of plane waves and the induced currents can thus be easily expressed as a weighted sum of pre-computed tabulated basis functions at $K + 1$ incidence angles.

$$\begin{bmatrix} \overline{\overline{Z}}_a & \overline{\overline{Z}}_b \\ \overline{\overline{Z}}_c & \overline{\overline{Z}}_d \end{bmatrix} \begin{bmatrix} \bar{j}^{(k,0)} \\ \bar{k}^{(k,0)} \end{bmatrix} = \begin{bmatrix} \bar{p}_{q,0}^{(k)} \\ \bar{0} \end{bmatrix},\ k = 0...K \tag{19}$$

where

$$\bar{p}_{q,0}^{(k)} = \begin{bmatrix} e^{-k_0 \phi_{in}^{(k)} \frac{Q}{2}\Delta x} \\ e^{-k_0 \phi_{in}^{(k)} \frac{Q-1}{2}\Delta x} \\ . \\ . \\ e^{k_0 \phi_{in}^{(k)} \frac{Q}{2}\Delta x} \end{bmatrix} \tag{20}$$

$$\phi_{in}^{(k)} = \frac{k\pi}{K},\ k = 0...K \tag{21}$$

B. Scattered field from group to group

Inserting the approximation from equation (14) into the expressions for $\overline{\overline{Z}}_{ij}$ and assuming the far-field form of the Hankel function, allow us to write the sub-matrix interaction between the groups as

$$\begin{aligned} \overline{\overline{Z}}_{ij} &= \begin{bmatrix} \overline{\overline{Z}}_{ij}^{(a)} & \overline{\overline{Z}}_{ij}^{(b)} \\ \overline{\overline{Z}}_{ij}^{(c)} & \overline{\overline{Z}}_{ij}^{(d)} \end{bmatrix} && (22)\\ &\simeq \begin{bmatrix} Z_{ll'}^{(a)} \bar{p}_{q,0} \bar{p}_{w,0}^T & Z_{ll'}^{(b)} \bar{p}_{q,0} \bar{p}_{w,0}^T \\ Z_{ll'}^{(c)} \bar{p}_{q,1} \bar{p}_{w,1}^T & Z_{ll'}^{(d)} \bar{p}_{q,1} \bar{p}_{w,1}^T \end{bmatrix} && (23) \end{aligned}$$

where $\overline{p}_{q,\alpha}$ and $\overline{p}_{w,\alpha}^{T}$, $\alpha = 0, 1$ are $N \times 1$ vectors given by:

$$\overline{p}_{q,\alpha} = \begin{bmatrix} e^{-k_\alpha \hat{r}_{ll'} \cdot \hat{t}_j \frac{Q}{2} \Delta x} \\ e^{-k_\alpha \hat{r}_{ll'} \cdot \hat{t}_j \frac{Q-1}{2} \Delta x} \\ . \\ . \\ e^{k_\alpha \hat{r}_{ll'} \cdot \hat{t}_j \frac{Q}{2} \Delta x} \end{bmatrix} \tag{24}$$

and

$$\overline{p}_{w,\alpha} = \begin{bmatrix} e^{-k_\alpha \hat{r}_{ll'} \cdot \hat{t}_i \frac{Q}{2} \Delta x} \\ e^{-k_\alpha \hat{r}_{ll'} \cdot \hat{t}_i \frac{Q-1}{2} \Delta x} \\ . \\ . \\ e^{k_\alpha \hat{r}_{ll'} \cdot \hat{t}_i \frac{Q}{2} \Delta x} \end{bmatrix} \tag{25}$$

Then, the far-field scattered from group G_j to group G_i is expressed as:

$$\overline{\overline{Z}}_{ij}\overline{x}_j = \begin{bmatrix} \overline{\overline{Z}}_{ij}^{(a)} & \overline{\overline{Z}}_{ij}^{(b)} \\ \overline{\overline{Z}}_{ij}^{(c)} & \overline{\overline{Z}}_{ij}^{(d)} \end{bmatrix} \begin{bmatrix} \overline{j}_j \\ \overline{k}_j \end{bmatrix} \tag{26}$$

$$\simeq \begin{bmatrix} Z_{ll'}^{(a)} \overline{p}_{q,0}\overline{p}_{w,0}^{T} & Z_{ll'}^{(b)} \overline{p}_{q,0}\overline{p}_{w,0}^{T} \\ Z_{ll'}^{(c)} \overline{p}_{q,1}\overline{p}_{w,1}^{T} & Z_{ll'}^{(d)} \overline{p}_{q,1}\overline{p}_{w,1}^{T} \end{bmatrix} \times \begin{bmatrix} \sum_{k=0}^{K} \left(\psi_j^{(k)} \overline{j}^{(k,0)} + \varphi_j^{(k)} \overline{j}^{(k,1)} \right) \\ \sum_{k=0}^{K} \left(\psi_j^{(k)} \overline{k}^{(k,0)} + \varphi_j^{(k)} \overline{k}^{(k,1)} \right) \end{bmatrix} \tag{27}$$

From equation (27), a similar derivation to equation (18) is applied and it can be shown that scattered field from group G_j to group G_i has the form of plane waves and the scattered field is a function of distance between centers of group G_j and group G_i, angle of scattered wave from group G_j ,$\hat{r}_{ll'} \cdot \hat{t}_j$ and angle of scattered wave to group G_i ,$\hat{r}_{ll'} \cdot \hat{t}_i$. Hence the right hand side of equation (6) can be expressed as a sum of plane waves and the induced currents can thus be interpolated from pre-computed tabulated basis functions at $K + 1$ incidence angles.

$$\begin{bmatrix} \overline{\overline{Z}}_a & \overline{\overline{Z}}_b \\ \overline{\overline{Z}}_c & \overline{\overline{Z}}_d \end{bmatrix} \begin{bmatrix} \overline{j}^{(k,0)} \\ \overline{k}^{(k,0)} \end{bmatrix} = \begin{bmatrix} \overline{p}_{q,0}^{(k)} \\ \overline{0} \end{bmatrix}, \; k = 0...K \tag{28}$$

$$\begin{bmatrix} \overline{\overline{Z}}_a & \overline{\overline{Z}}_b \\ \overline{\overline{Z}}_c & \overline{\overline{Z}}_d \end{bmatrix} \begin{bmatrix} \overline{j}^{(k,1)} \\ \overline{k}^{(k,1)} \end{bmatrix} = \begin{bmatrix} \overline{0} \\ \overline{p}_{q,1}^{(k)} \end{bmatrix}, \; k = 0...K \tag{29}$$

IV. RESULTS

In this section, the accuracy and computational complexity of the proposed TIM method is evaluated. The fields generated by the TIM are compared with measurement data. Moreover, a comparison of the TIM with CBFM is performed in terms of accuracy and processing time. Two distinct types of terrain profiles, slightly rough (hilly) profiles and very rough (mountainous) profiles were considered to demonstrate the robustness of the proposed method. Forward scattering was assumed and the terrain was modelled as dry soil which is a dielectric with $\varepsilon_r = 2.8$ and loss tangent $\tan\delta = 0.07$. The transmitting antenna is a dipole with the transmitting power $10W$. The computer used in the simulation was a Dell Precision Workstation 670 which has a Xeon 3.6 GHz CPU and 3GB of memory. The simulation environment was Matlab 7.8.0.

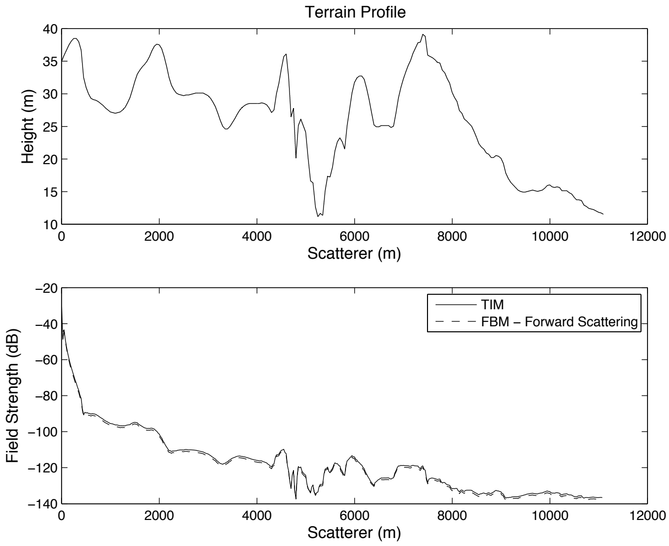

Figure 1. Fields generated by the TIM and FBM-Forward Scattering. Terrain profile: Hjorring - Denmark.

A. Accuracy of TIM

In order to illustrate the accuracy of the TIM, the total field generated by the TIM is compared to that generated by an exact solution assuming forward scattering. This can be readily, if slowly, computed assuming using back substitution. Two distinct types of terrain profiles, slightly rough (hilly) surfaces and very rough (mountainous) surfaces were considered. In the case of slightly rough surfaces, the propagation over Hjorring terrain which has the size of $11km$ is considered in figure (1). The source was located at (0.0m,45.4m) and radiated TM^z polarised fields at $144MHz$. The receiving antenna is located at the height of 2.4m above the terrain profile. Groups of size $50m$ were used to implement TIM.

Figure (2) illustrates the total field over a very rough terrain profile which has size of $3.8km$. Groups of size $10m$ were used to implement TIM. As the surfaces become more mountainous, the group size is narrowed to reduce the errors inherent in the TIM approximations. Figure (1) and (2) show that the fields computed by the TIM are in good agreement the field computed by the FBM - Forward Scattering.

B. Performance comparison between the TIM and CBFM

In this section, a comparison between the TIM and the CBFM-FBM [2], [3] is performed in terms of accuracy and processing time. The group size of CBFM is $50m$. In the CBFM-FBM, two basis functions, primary basis functions (PBFs) and secondary basis functions (SBFs), were used to construct the currents on the surface [3]. Figure (3) shows that

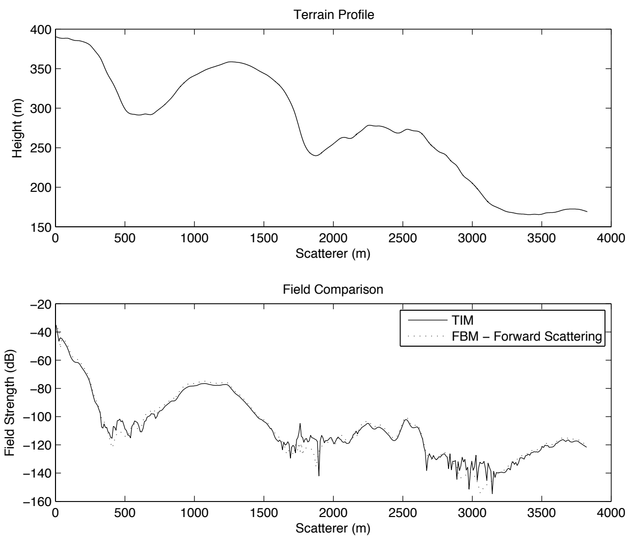

Figure 2. Fields generated by the TIM and FBM-Forward Scattering. Terrain profile: very rough surface.

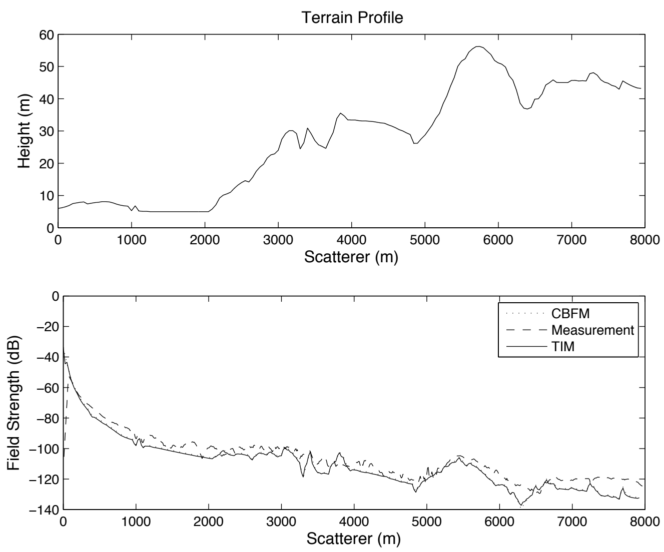

Figure 3. Fields generated by TIM, CBFM and measurement data. Terrain profile: Hadsund - Denmark.

the fields computed by the TIM matches the field computed by CBFM-FBM. Moreover, a comparison versus measured field strength is also given in figure (3).

A comparison in computational complexity was performed by evaluating processing time of TIM, CBFM and FBM - Forward Scattering. Table (1) and (2) shows that the TIM is dramatically faster than CBFM-FBM and FBM-Forward Scattering. In the TIM, the basis functions were pre-calculated once and then used for the calculation of the electric and magnetic currents on the surface of the terrain profile. The FAFFA is then invoked to speed up the interaction between groups. In contrast, the CBFM-FBM involves the calculation of independent basis functions for *each* group of terrain profile. Therefore, the TIM has a much lower computational complexity than CBFM-FBM, although the CBFM has wider

Table I
PROCESSING TIME OF TIM, CBFM AND FBM-FORWARD SCATTERING. TERRAIN PROFILE: HADSUND. DISCRETIZATION SIZE: $\frac{\lambda}{4}$. SCATTERER: PEC.

Frequency	Number of unknowns	Run time (seconds)		
(MHz)		TIM	CBFM	FBM-FS
144	20353	2.03	289	298
435	81408	2.04	1454	4687
970	162816	2.03	-	19331

Table II
PROCESSING TIME OF THE TIM AND FBM-FORWARD SCATTERING. TERRAIN PROFILE: HADSUND. DISCRETIZATION SIZE: $\frac{\lambda}{8}$. SCATTERER: DIELECTRIC.

Frequency	Number of unknowns	Run time (seconds)	
(MHz)		TIM	FBM-FS
144	81408	7.05	5134
435	325632	7.18	82250
970	655360	6.93	-

applicability.

V. CONCLUSIONS

In this paper, an extension of the TIM to compute the electromagnetic scattering from lossy dielectric terrain profiles was presented. The accuracy, complexity and performance of the TIM has been evaluated and compared to those of FBM-Forward Scattering and CBFM-FBM. In addition, the similarities and differences between the TIM and CBFM-FBM has been analyzed and explained. The numerical analysis demonstrates a better performance in terms of run time of the TIM when compared to FBM-Forward Scattering and the CBFM.

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