The strictly non-blocking condition for three-stage networks

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Abstract

A criterion for a three-stage network to be strictly non-blocking is presented which is very general in its application. The criterion distinguishes between channel grouping and link speedup as methods of increasing the bandwidth available to calls. It may be applied to both circuit-switched and packet-switched networks. The non-blocking conditions for various networks are shown to be special cases of the condition presented here.

1. INTRODUCTION

The condition for a single-rate, single-channel, three-stage circuit switch to be non-blocking is well known [1]. A number of authors have sought to extend this result to switches with multiple rates, or multiple channels, and to packet switching [2-9]. We present a condition for a three-stage network to be strictly non-blocking, of which existing conditions are a special case. The network considered is shown in Fig. 1.

2. THE GENERAL CONDITION

2.1. Symmetric networks

The non-blocking condition for a symmetric three-stage network of the type in Figure 1(a), where m is the number of modules in the intermediate stage, shall now be derived, under the following assumptions:

i. The input (output) modules of the switch have n inputs (outputs) and Sm outputs (inputs).

ii. The input and output of the switch links operate at a rate of v bits/sec.

iii. The intermediate links operate at a rate of rv bits/sec.

iv. The intermediate links are in trunk groups of S links each.

v. The call rate u may occupy a continuous or discrete set of values Z in the range $0 < u_{min} \le u \le u_{max} \le v$.

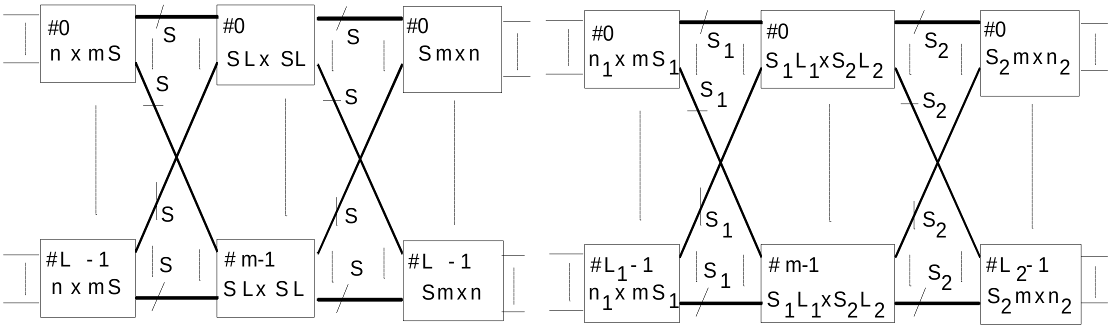
vi. It is possible to offer 100% load to each input module, i.e., a load of nv bits/s.

vii. Traffic from one call can be assigned a route on only one link in a trunk group, i.e., traffic from one call may not be split across links in a group.

viii. Blocking occurs on a link when the offered load exceeds the link capacity.

ix. The switch modules are non-blocking.

The value of m required to ensure that this switch is strictly non-blocking for circuit-switched traffic shall now be obtained.



(a) symmetric (b) asymmetric

Figure 1: Three-stage switches with channel groups and link speed-up.

Consider the case where a call of rate u is to be placed from input module 1 (IM_1) to output module 1 (OM_1), and where this will result in a 100% load on IM_1 and OM_1. The existing load on both switch modules is thus nv-u.

Each intermediate link of the switch has a capacity of rv. The minimum aggregate rate of existing traffic on such a link which results in that link being unavailable to a call of rate u shall be denoted $F(u)$. Let Ω be the set of non-zero values which the aggregate rate of traffic on an intermediate link can occupy. Then

$$F(u) = \min_{x \in \Omega} \{x \mid x + u > r\,v\}.$$

In the worst case, the traffic from an input module to an intermediate module will be evenly shared among all the links in the corresponding trunk group. Hence the minimum level of traffic between the two modules which results in a call of rate u being blocked is $SF(u)$.

Traffic from IM_1 to OM_1 will, in the worst case, be distributed such that the maximum possible number of trunk groups will carry traffic of an aggregate rate of $SF(u)$. The total number of intermediate modules which cannot accept a call of rate u from IM_1 is then

$$\left\lfloor \frac{nv-u}{S.F(u)} \right\rfloor.$$

A similar analysis shows that, in the worst case, the number of intermediate modules which cannot route a call of rate u to OM_1 is

$$\left\lfloor \frac{nv-u}{S.F(u)} \right\rfloor$$

The maximum number of intermediate modules is unavailable to a call of rate u if the intermediate modules which block its call on the input side, and those which block on the output side of the module, are distinct. Hence, blocking is impossible for a call of rate u if

$$m > 2\left\lfloor \frac{nv-u}{S.F(u)} \right\rfloor .$$

Hence, no blocking can occur for any call if

$$m > 2\max_{u \in Z}\left\lfloor \frac{nv-u}{S.F(u)} \right\rfloor.$$

2.2. Asymmetric switches

The non-blocking condition for the asymmetric switch in Figure 1(b) is

$$m > \max_{u \in Z}\left(\left\lfloor \frac{n_1 v-u}{S_1.F_1(u)} \right\rfloor + \left\lfloor \frac{n_2 v-u}{S_2.F_2(u)} \right\rfloor \right),$$

where $F_1(u) = \min_{x\in\Omega_1} \{x \mid x + u > r_1 v\}$, $F_2(u) = \min_{x\in\Omega_2} \{x \mid x + u > r_2 v\}$, and Ω_1 (Ω_2) is the set of values which the aggregate rate of traffic on an intermediate link entering (leaving) the intermediate stage can occupy. The proof of this result follows by a straightforward extension of that for the symmetric case.

3. SPECIAL CASES OF THE NON-BLOCKING CONDITION

3.1. Single rate switch with no channel grouping or link speedup

Suppose $S = r = 1$ and $u_{min} = u_{max} = v$. Then $Z = \Omega = \{v\}$. Since $F(v) = v$, it follows that

$$m > 2\left\lfloor \frac{nv - v}{v} \right\rfloor,$$

i.e., $m > 2(n-1)$ .This is the result first obtained by Clos [1].

3.2. Single rate switch with channel grouping and link speedup

Suppose $v = f_i u_b$ where f_i is an integer. Also $r = f_0 / f_i$ where f_0 is an integer and $f_0 > f_i$. Then $Z = \{u_b\}$ and $\Omega = \{ku_b , 0 < k \le f_0\}$. Each intermediate link can support f_0 calls. Hence

$F(u_b) = \min_{0<j\le f_0} \{j u_b \mid j u_b + u_b > f_0 u_b\}$, i.e., $F(u_b) = f_0 u_b$. Consequently

$$m > 2\left\lfloor \frac{nf_i u_b - u_b}{S.f_0.u_b} \right\rfloor, \text{ i.e., } m > 2\left\lfloor \frac{nf_i - 1}{S.f_0} \right\rfloor.$$

This is the strictly non-blocking condition obtained by Jajszczyk [2].

3.3. Multi-rate switch with channel grouping and link speedup

This case is the extension of the above case to multi-rate traffic, where $Z = \{ku_b, 0 < k \le k_m\}$, with $k_m \le f_i$. Here

$$F(k u_b) = \min_{0<j\le f_0} \{j u_b \mid j u_b + k u_b > f_0 u_b\}$$

so

$$F(k u_b) = (f_0 - k+1) u_b.$$

Therefore

$$m > 2 \max_{0<k\le k_m} \left\lfloor \frac{nf_i u_b - ku_b}{S.(f_0 - k + 1).u_b} \right\rfloor,$$

i.e.,

$$m > 2 \max_{0<k\le k_m} \lfloor g(k) \rfloor,$$

where

$$\gamma(k) = \frac{nf_i u_b - ku_b}{S.(f_0 - k + 1).u_b}.$$

It may readily be shown that $\gamma(k)$ is a monotonic increasing function of k, provided that $nf_i > f_0+1$, which will be the case for any practical switch. Hence

$$\max_{0<k\le k_m} \lfloor g(k) \rfloor = \left\lfloor \frac{nf_i - k_m}{S.(f_0 - k_m + 1)} \right\rfloor.$$

Therefore

$$m > 2\left\lfloor \frac{nf_i - k_m}{S.(f_0 - k_m + 1)} \right\rfloor.$$

3.4. Multi-rate switch with no channel grouping or speedup

The result in section 3.3 reduces, in the special case where $f_0 = f_i$ (i.e., where $r = 1$ and $S = 1$) to

$$m > 2\left\lfloor \frac{nf_i - k_m}{f_i - k_m + 1} \right\rfloor .$$

This is the result obtained by Niestegge [3] for a multi-rate network without internal speed-up.

3.5. Variable rate switch with link speedup but no channel grouping

Suppose $S = 1$ and $Z = \{u \mid u_{min} \le u \le u_{min}\}$. Hence $\Omega = \{x \mid u_{min} \le x \le rv\}$.
Therefore $F(u) = \min_{x\in\Omega}\{x \mid x + u > rv\}$.

Consider first the case where $u_{min} = 0$. In this case
$F(u) = \lim_{e\to 0^+}\{rv - u + e\} = rv - u$.

However, if $u_{min} > 0$, it is necessary that $F(u) \ge u_{min}$.
Hence $F(u) = max\,(rv - u\,,\,u_{min})$. Thus

$$m > 2 \max_{u_{\min}\le u\le u_{\max}} \left\lfloor \frac{nv - u}{\max(rv - u, u_{\min})} \right\rfloor .$$

This result can be shown to be the same as that obtained by Melen and Turner [4]. It follows that an unstated assumption of Melen and Turner is that the call rate u can occupy a continuum of values in the range from u_{min} to u_{max}.

4. NON-BLOCKING CONDITIONS FOR ATM NETWORKS

4.1. Time scales for path allocation

The above conditions need to be modified before being applied to packet-switched networks or ATM networks. Two different sets of results must be obtained, because of the two distinct strategies possible for performing routing in three-stage ATM switches.

Multiple paths from source to destination are available in a three-stage switch. Hence a path allocation algorithm must be employed to select among the available paths. A distinction is made between two time scales over which path allocations may be performed:

- Call level;
- Cell level.

Cell-level path allocation has been proposed in the Growable packet switch [10] and in our channel-grouped switch [11]. Most other proposals for three-stage ATM switches have featured call-level path allocation.

4.2 .Call-level path allocation

The condition already developed for circuit-switching can be applied to the call-level case. The bit-rate of a call can vary continuously from zero to the peak rate u. Hence, the appropriate formula is

$$m > 2\max_{0\le u\le v}\left\lfloor \frac{nv - u}{S.(rv - u)} \right\rfloor ,$$

i.e.,

$$m > 2\left\lfloor \frac{n-1}{S.(r-1)} \right\rfloor .$$

This result has been obtained by Melen & Turner [4] in the case where $S = 1$, and is valid for $r > 1$. It indicates that a speed up of the internal links is essential if a three-stage packet switch with a call-level path allocation algorithm is to be non-blocking. Otherwise even the smallest level of background traffic is sufficient to block a call at the peak rate u; in the worst case, all intermediate links will have some background traffic, however light.

Note that it has been assumed that all cells on a virtual circuit are routed on the same *channel*, and not just on the same *channel group* (assumption (*vii*) of Section 2.1) . If this assumption does not hold, no distinction is necessary between channel speed-up (r) and group size (S). The condition on m then becomes

$$m > 2\left\lfloor \frac{n-1}{Sr-1} \right\rfloor .$$

The term 'blocking' requires some interpretation when used in the context of a call-level routing algorithm. Its meaning depends on what is defined by the call rate u. One approach is to consider u to be the *equivalent bandwidth* (more strictly, the equivalent bit-rate) of the call. This is the amount of bandwidth which must be available to the call, if excessive cell loss is to be avoided. Inherent in this approach is the assumption that the equivalent bandwidth of the aggregate of calls sharing a link is a linear function of the equivalent bandwidths of the individual calls. This problem has been considered by Svinnset [5].

4.3. Cell-level path allocation

In this context, the conditions applicable when using call-level path allocation are not valid, since the cells belonging to a given virtual circuit are unlikely to be routed via the same intermediate switch module. In this case, a three-stage ATM switch more closely resembles a single-rate circuit switch since, for the duration of each time slot, each input is either inactive or transmitting data at the full ATM rate.

The relevant non-blocking condition for a symmetric single-rate three stage circuit switch is obtained where $Z = \{v\}$, and $\Omega = \{kv , 0 < k \le r\}$. (It is assumed that r is an integer. Otherwise r should be replaced by $\lfloor r \rfloor$ in the following calculations, since the additional bandwidth on each intermediate link cannot be used.)

Hence the non-blocking condition is a special case of that obtained in Section 3.2, where $f_i = 1$ and $f_0 = r$. i.e.,

$$m > 2\left\lfloor \frac{n-1}{S.r} \right\rfloor .$$

An ATM switch satisfying this condition can route n cells to each output module in each time slot. However, in packet switching, there is no longer a one-to-one correspondence between the number of output ports n of an output module and the number of requests for that output module. Thus, if all cells are to be guaranteed passage through the intermediate stage of the switch, it must be dimensioned so that *all* cells can be routed to the same output module. It may be shown that m must be chosen such that

$$m > \left\lfloor \frac{nL}{S.r} \right\rfloor ,$$

where L is the number of input modules.

A switch dimensioned using this criterion can route all arriving packets to the requested output modules, regardless of the pattern of such requests. Thus it exhibits zero loss in the input and intermediate stages. Such a switch would be prohibitively expensive. In practice, a finite probability of loss can be accepted, for which the required value of *m* will be considerably reduced [12,13].

5. MULTI-SLOT SWITCHING REQUIRING CONSECUTIVE TIME SLOTS

The non-blocking condition for multi-slot switching leads to a larger value for *m*, in general, because blocking can occur even when a link has idle capacity. We now obtain the condition on *m* for the switch to be non-blocking, under the following assumptions:

i. The input (output) modules of the switch have *n* inputs (outputs) and *Sm* outputs (inputs).

ii. A time-slotted operation is used, with framing.

iii. The frame length is f_i time slots (for input and output links) or f_0 time slots (for intermediate links); the frame repetition rate is constant throughout the switch.

iv. The input and output links of the switch operate at a rate of $v = f_i u_b$ bits/sec.

v. The intermediate links operate at a rate of $rv = f_0 u_b$ bits/sec.

vi. The intermediate links are in trunk groups of *S* links each.

vii. All calls are of a bit rate drawn from the set $Z = \{\, k\, u_b,\ 0 < k \le k_m\}$, with $k_m \le f_i$.

viii. It is possible to offer 100% load to each input module, i.e., a load of *nv* bits/s.

ix. A call of rate $k\, u_b$ can be placed on a link only if there are *k consecutive* free time slots in the frame.

x. The switch modules are non-blocking; the contents of any time slot of an input frame may be transferred to any time slot of the corresponding frame of any output.

Note that assumption ix. implies that blocking can occur on a link even if it has sufficient free capacity to accept the call, if the capacity is fragmented across the frame. Kabacinski [9] has shown that, where the frame length on a link is f_0 time slots, a call of rate $k\, u_b$ can be blocked if the existing traffic on the link has a bit rate of

$$F(ku_b) = \left\lceil \frac{f_0}{k} \right\rceil u_b$$

This corresponds to the worst case traffic pattern shown in Figure 2(a). The number of calls of rate u_b required to ensure that no *k* consecutive time slots are available is the largest integer value *n* such that

$$(n\text{-}1)k+1 \le f_0 .$$

Evidently

$$n = \left\lfloor \frac{f_o - 1}{k} \right\rfloor + 1 = \left\lceil \frac{f_o}{k} \right\rceil .$$

This result is valid if it is assumed that calls are allowed to straddle the frame boundary, i.e., if, for example, a call of rate $3\, u_b$ can occupy time slots f_0, 1 and 2 of a frame.

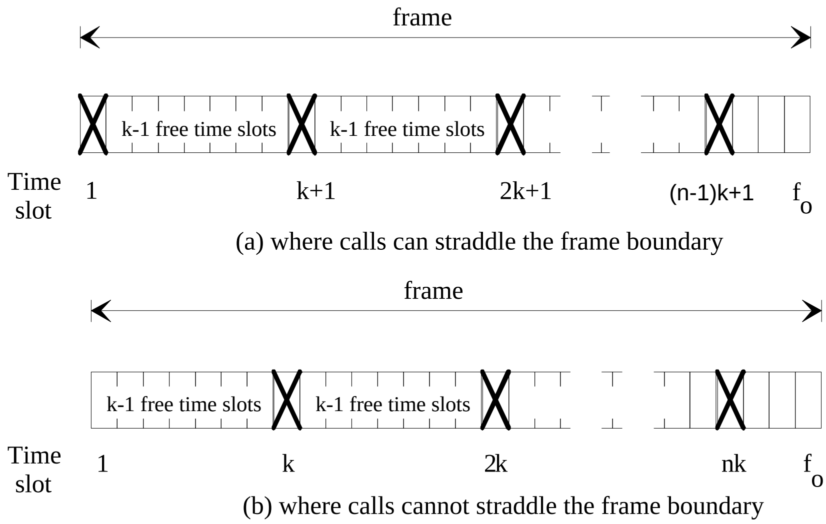


Figure 2: Blocking on a link.

Consider the case where f_0 = 8 and k = 3. The inequality above indicates that at least 3 calls of rate u_b must be present to block a call of rate 3 u_b. However, if boundary straddling is not allowed, blocking can occur with only two calls of rate u_b present, as shown in Figure 3. A call of rate 3 u_b can be placed in the shaded time slots only if boundary straddling is allowed.

When boundary straddling is not permitted, the worst case traffic pattern is as shown in Figure 2 (b). The number of calls of rate u_b required to ensure that no k consecutive time slots are available is the largest integer value n such that

$nk \le f_0$.

Evidently

$$n = \left\lfloor \frac{f_0}{k} \right\rfloor .$$

To summarise, a call of rate k u_b can be blocked from access to a link of rate f_0 u_b (with a frame length of f_0 time slots) if the aggregate bit rate of calls already using the link is greater than or equal to

$$F(ku_b) = \begin{cases} \left\lceil \frac{f_0}{k} \right\rceil & \text{(with straddling of frame boundaries)} \\ \left\lfloor \frac{f_0}{k} \right\rfloor & \text{(without straddling of frame boundaries)} \end{cases}$$

The non-blocking condition can now be obtained in a manner similar to that used in Section 3.3. The required value of m is

$$m > 2 \max_{0<k\le k_m} \left\lfloor \frac{nf_i u_b - ku_b}{S.F(ku_b)} \right\rfloor ,$$

i.e., $m > 2 \max_{0<k\le k_m} \lfloor g(k) \rfloor$,

where $g(k) = \dfrac{nf_i u_b - ku_b}{S.F(ku_b)}$.

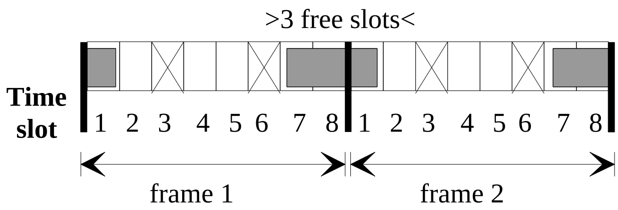


Figure 3: An example of time slot assignment

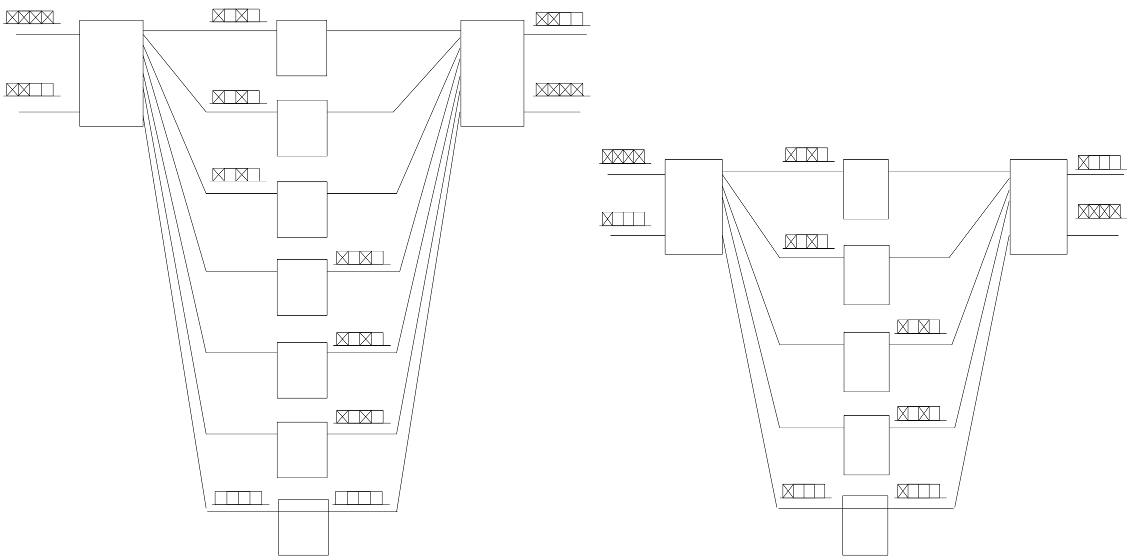
The function $\gamma(k)$ is not, in general, a non-decreasing function of k, so that it does not, in general, follow that the worst case of blocking occurs when a call of rate k_m u_b is being placed. Consider the case where $f_i = f_0 = 4$, $S = 1$, $n = 2$, and $k_m = 3$, with boundary straddling permitted. As many as six modules in the intermediate stage can be blocked for a call of rate of rate 2 u_b , as shown in Figure 4(a). However, as illustrated in Figure 4(b), at most four of the intermediate stage modules can be blocked when a call of rate 3 u_b is placed.

It follows that it is necessary to calculate the value of $\gamma(k)$ for all values of k in the range $0 < k \le k_m$, in order to find the value of m which ensures that the switch is non-blocking. Hence, Kabacinski's condition for a three-stage switch to be non-blocking, i.e., that

$$m > 2\left\lfloor \frac{nf_i - k_m}{S.\left\lceil \frac{f_0}{k_m} \right\rceil} \right\rfloor,$$

should be rewritten as

$$m > 2 \max_{0<k\le k_m} \left\lfloor \frac{nf_i - k}{S.\left\lceil \frac{f_0}{k} \right\rceil} \right\rfloor.$$



(a) call rate is 2 u_b. **(b) call rate is 3 u_b**

Figure 4: Number of intermediate stage modules blocked, in the worst case, for a frame length of four time slots.

If boundary straddling is not allowed, this becomes

$$m > 2 \max_{0<k\leq k_m} \left\lceil \frac{nf_i - k}{S.\left\lfloor \frac{f_0}{k} \right\rfloor} \right\rceil .$$

The values of *m* required, as given by the above inequalities will, in general, be much larger than those indicated by the results in Section 3.3. The latter will be valid only if, every time a call terminates, the time slots are rearranged so that the idle time slots remain contiguous, or if the switch can allocate a call to non-consecutive time slots. Hence, the switch requires either a high intermediate stage bandwidth, or a complex control algorithm, or both.

6. CONCLUSIONS

A criterion for a network to be strictly non-blocking has been presented which is applicable to a very broad range of three-stage networks. The criterion makes a distinction between channel grouping and link speedup as methods of increasing the bandwidth available to calls. It may be applied to both circuit-switched and packet-switched networks. It has been shown that the non-blocking conditions for various networks are special cases of the condition presented here.

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