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Declaration

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Programme: MENC (Masters Electronic Systems)

Module Code: EE506

Assignment Title: EE506 Assignment

Submission Date: 24-April-2015

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

I have identified and included the source of all facts, ideas, opinions, viewpoints of others in the assignment references. Direct quotations from books, journal articles, internet sources, module text, or any other source whatsoever are acknowledged and the sources cited are identified in the assignment references.

I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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By signing this form or by submitting this material online I confirm that this assignment, or any part of it, has not been previously submitted by me or any other person for assessment on this or any other course of study.

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Name: Ankit Bose

Date: 24-April-2015

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EE506 Assignment

Q-1)

1.1)
$$\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e}$$

The symbols in above set of equations mean the follows:—

I = bias current applied to the laser.
A = Coefficient of gain
τ_e = carrier lifetime
τ_p = photon lifetime
N = carrier number
P = photon number
β = Spontaneous emission coupling
N_0 = carrier number at transparency
$\frac{dN}{dt}$ = Rate of equation for N

$\frac{dP}{dt}$ = Rate of equation for P

1.2) To calculate the threshold current, we assume the rate equations to be in steady state condition i.e. $\frac{d}{dt} = 0$ no time dependancy. Since the optical gain is linear with pumping, i.e. the current injected and therefore the carrier number. It is possible to approximate the gain as a linear function of the carrier.

$$G = A(N - N_0)$$

The expression for threshold current is achieved by considering that $GP >> \beta \frac{N}{\tau_e}$ where GP is stimulated emission & $\beta \frac{N}{\tau_e}$ is spontaneous emission.

The spontaneous emission is neglected & Gain becomes equal to the losses. i.e. $A(N - N_0) = \frac{1}{\tau_p}$

$$\therefore N = N_0 + \frac{1}{A\tau_p}$$

Using the above value N in carrier rate equation

$$\frac{I}{e} - \frac{N}{\tau_e} = A(N - N_0)P \Rightarrow \cancel{\frac{I}{e}}\ \frac{I}{e} - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right) = \frac{P}{\tau_p}$$

$\therefore$ P can be expressed as $P = \frac{\tau_p}{e}(I - I_s)$ where I_s is the threshold current with

$$I_s = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

By inserting the given values in the above equation, we get

$I_s = \underline{14.39 \text{ mA}}$

1.3) Below the threshold current, the number of carriers keeps increasing with the increase in bias current according to this expression:

$$N = \frac{\tau_e}{e} I$$

Above the threshold current, the number of carriers is clamped at a value $N_0 + \frac{1}{A\tau_p}$ when the gain is equal to the losses.

As the bias current increases, the carrier number is fixed and the gain will not increase. So, the excess of carriers ~~and~~ are directly used for stimulated emission.

2.1) For small signal analysis on steady state ~~quantities~~ quantities $\bar{N}, \bar{P}, \bar{I}$, we introduce a perturbation of the current δI, which results in perturbation of the carrier number δN and photon number δP. Instead of I we use $\bar{I} + \delta I$, of N, $\bar{N}+\delta N$, of P, $\bar{P} + \delta P$ & the rate equation becomes:

$$\frac{d\bar{N}+\delta N}{dt} = \frac{\bar{I}+\delta I}{e} - A(\bar{N}+\delta N-N_0)(\bar{P}+\delta P) - \frac{\bar{N}+\delta N}{\tau_e}$$

$$\frac{d\bar{P}+\delta P}{dt} = A(\bar{N}+\delta N-N_0)(\bar{P}+\delta P) - \frac{\bar{P}+\delta P}{\tau_p} + \beta\frac{\bar{N}+\delta N}{\tau_e}$$

In order to develop these equations, the second order expressions i.e. $\delta N, \delta P$ are neglected and set to zero. ~~Therefore, the equation for perturbations becomes: Also, it is~~
Since the analysis is carried out above threshold i.e. $A(\bar{N}-N_0) = \frac{1}{\tau_p}$, the above equations can be re-written as:

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \left(A\bar{P} + \frac{1}{\tau_e}\right)\delta N - \frac{\delta P}{\tau_p}$$

$$\frac{d(\delta P)}{dt} = A\bar{P}\,\delta N$$

These equations can be expressed in the matrix form as:

$$\begin{pmatrix} \frac{d(\delta N)}{dt} \\ \frac{d(\delta P)}{dt} \end{pmatrix} = \begin{pmatrix} -\left(A\bar{P}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

$$\therefore M = \begin{pmatrix} -\left(A\bar{P}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix}$$

2.2) From the above equation in 2.1, we found that

$$\begin{pmatrix} \frac{d\,\delta N}{dt} \\ \frac{d\,\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -\left(A\bar{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$

where $M = \begin{pmatrix} -\left(A\bar{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix}$

2.3) To determine the value of perturbations $\delta\tilde{P}$ and $\delta\tilde{N}$ as a function of $\delta\tilde{I}$, the equation in Fourier domain can be written as:

$$\begin{pmatrix} j\Omega\,\delta\tilde{N} \\ j\Omega\,\delta\tilde{P} \end{pmatrix} = \begin{pmatrix} -\left(A\bar{P}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix} + \begin{pmatrix} \delta\tilde{I}/e \\ 0 \end{pmatrix}$$

If we group the expressions together, the equation becomes

$$\begin{pmatrix} j\Omega+\left(A\bar{P}+\frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\bar{P} & j\Omega \end{pmatrix} \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix} = \begin{pmatrix} \delta\tilde{I}/e \\ 0 \end{pmatrix}$$

We could consider:

$$M = \begin{pmatrix} j\Omega+\left(A\bar{P}+\frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\bar{P} & j\Omega \end{pmatrix}$$

$$\therefore \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix} = M^{-1} \begin{pmatrix} \delta\tilde{I}/e \\ 0 \end{pmatrix}$$

If M is expressed as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then,

$$M^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\therefore M^{-1} = \frac{1}{-\Omega^2 + \frac{A\bar{P}}{\tau_p} + j\Omega\left(A\bar{P}+\frac{1}{\tau_e}\right)} \begin{pmatrix} j\Omega & -\frac{1}{\tau_p} \\ A\bar{P} & j\Omega+\left(A\bar{P}+\frac{1}{\tau_e}\right) \end{pmatrix}$$

$$\omega_R^2 = \frac{A\bar{P}}{\tau_p} \quad \text{and} \quad \frac{2}{\tau_R} = A\bar{P} + \frac{1}{\tau_e}$$

ω_R is the relaxation oscillation and $\frac{2}{\tau_R}$ is the damping coefficient.

$$\therefore \cancel{\delta\tilde{N} = \frac{j\Omega}{\omega^2 - \Omega^2}}$$

$$\therefore \quad \delta\tilde{N} = \frac{j\Omega}{-\Omega^2 + \frac{A\bar{P}}{\tau_p} + j\Omega\left(A\bar{P} + \frac{1}{\tau_e}\right)} \frac{\delta\tilde{I}}{e}$$

$$\delta\tilde{N} = \frac{j\Omega}{-\Omega^2 + \omega_R^2 + j\Omega\frac{2}{\tau_R}} \frac{\delta\tilde{I}}{e}$$

$$\therefore \quad \delta\tilde{P} = \frac{A\bar{P}}{-\Omega^2 + \frac{A\bar{P}}{\tau_p} + j\Omega\left(A\bar{P} + \frac{1}{\tau_e}\right)} \frac{\delta\tilde{I}}{e}$$

$$\delta\tilde{P} = \frac{\omega_R^2 \tau_p}{-\Omega^2 + \omega_R^2 + j\Omega\frac{2}{\tau_R}} \frac{\delta\tilde{I}}{e}$$

3) $$\frac{S}{N} = \frac{\left(\frac{\eta e}{hc/\lambda}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{hc/\lambda}\right)P_s + \frac{4kT}{R_c}\right)\Delta f}$$

3.1) The given values are :-

$\eta = 0.85$ $\qquad k = 1.38\times10^{-23}\ J/K$

$e = 1.6\times10^{-19}C$ $\qquad T = 300K$

$h = 6.62\times10^{-34}\ Js$ $\qquad R_c = 50\Omega$

$c = 3\times10^{8}\ m/s$ $\qquad \Delta f = 20\times10^{9}\ Hz$

$\lambda = 1.55\times10^{-6}\ m$ $\qquad P_s = -3dBm$

Changing P_s to linear scale

$\therefore P_s = 10^{-3/10} = 0.5\times10^{-3}\ W$

$\rightarrow \left(\frac{\eta e}{hc/\lambda}\right)^2 P_s^2$

$$= \left(\frac{0.85\times1.6\times10^{-19}\times1.55\times10^{-6}}{6.62\times10^{-34}\times3\times10^{8}}\right)^2 (0.5\times10^{-3})^2$$

$$= (1.06)^2\times(0.5\times10^{-3})^2$$

$$= 0.2809\times10^{-6}$$

$\rightarrow \frac{4kT}{R_c}$

$$= \frac{4\times1.38\times10^{-23}\times300}{50}$$

$$= 33.12\times10^{-23}$$

$\rightarrow 2e\left(\frac{\eta e}{hc/\lambda}\right)P_s$

$$= 2\times1.6\times10^{-19}\times(1.06)\times0.5\times10^{-3}$$

$$= 1.696\times10^{-22}$$

$$\therefore \frac{S}{N} = \frac{0.2809\times10^{-6}}{(1.696+3.312)10^{-22}\times20\times10^{9}}$$

$$= 0.028045\times10^{6}$$

$$= 10\log(0.028045\times10^{6})$$

$$= 44.47\ dB$$

3.2) At temperature $0^\circ C$ i.e. 273 K,

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{hc/\lambda}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{hc/\lambda}\right)P_s + \frac{4kT}{R_c}\right)\Delta f}$$

$$\rightarrow \frac{4kT}{R_c}$$

$$= \frac{4 \times 1.38 \times 10^{-23} \times 273}{50}$$

$$= 30.13 \times 10^{-23}$$

Plugging values from the ~~pre~~ previous part of the question:

$$\therefore \frac{S}{N} = \frac{0.2809 \times 10^{-6}}{(1.696 + 3.013)\, 20 \times 10^{9} \times 10^{-22}}$$

$$= 0.02982 \times 10^{6}$$

$$= 10 \log \left(0.02982 \times 10^{6}\right)$$

$$\therefore \frac{S}{N} = 44.74\ dB$$

3.3) From 3.1 & 3.2 questions, we note that
at 300K, SNR = 44.47dB
at 273K, SNR = 44.74dB
with decreasing temperature, SNR is increasing.
∴ with ~~more~~ increase in Temperature, the signal to noise ratio will decrease.

3.4) From the SNR expression, Noise i.e. $N = \left(2e\left(\frac{\eta e}{hc/\lambda}\right)P_s + \frac{4kT}{R_L}\right)\Delta f$

where Δf is the bandwidth. If the bandwidth increases, the noise will increase.
∴ If the bandwidth increases, the signal to noise ratio will decrease.

4) τ_r (radiative) = 50ns
τ_{nr} (non-radiative) = 75ns

$\therefore$ the internal efficiency $\eta_{int} = \dfrac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}}$

$$= \frac{1/50}{1/50 + 1/75}$$

$$= 0.6$$

$\lambda = 450nm$

$I = 300mA$

$c = 3\times10^8 m/s$

$h = 6.62\times10^{-34} Js$ $\quad e = 1.6\times10^{-19}$

$\therefore$ the external efficiency $\eta_{ext} = \dfrac{P/h\nu}{I/e}$

For extraction efficiency, we know that:

$$\eta = \eta_{int} \times K \times \left[1 - \left(\frac{n-1}{n+1}\right)^2\right] \frac{2\pi(1-\cos\theta_c)}{4\pi}$$

where η_{int} is the internal efficiency i.e. 0.6

K is absorption loss

n is refractive index i.e. 3.6

θ_c is critical angle.

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right)$$

$$= 16.12.$$

Given that semiconductor material is not light absorbing.

$\therefore K = 1$

$$\therefore \eta = 0.6 \times 1 \times \left[1 - \left(\frac{3.6-1}{3.6+1}\right)^2\right] \frac{2\pi(1-\cos(16.12))}{4\pi}$$

$$= 0.0077$$

$$= 0.77\%$$