DCU | **Dublin City University**
School of Electronic Engineering

Coursework Submission Cover Sheet

Student No	13210818	**Degree Scheme**	22.00%
Student Name	Sari Al Gaidy	**Year**	2015
Module	EE559	**Lecturer**	Dr. Pascal Landais
Title	Semester two solution assignment 2015	**Hours spent on this exercise**	25 hours

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Signed: **Date:** 24/04/2015

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Comments:

Q1:.

1.1:

They describe the time evolution of the carrier and photonic numbers.

P = The Photon number.

N = The carrier number.

A = The gain coefficient.

I = The value of the bias current applied to the laser.

τ_e = The carrier lifetime.

τ_p = The Photon lifetime.

β = The spontaneous emission coupling.

1.2

To get the threshold current, the rate equation must be used in steady state, which mean there is no time variation. $\frac{d}{dt}=0$. At threshold current, the spontaneous emission is execluded. ~~and gain~~

gain = losses.

$A(N-N_0)=\frac{1}{\tau_p}$. carrier number at threshold $=(N_0+\frac{1}{A\tau_p})$, which is used in the equation: $\frac{I}{e}-A(N-N_0)P-\frac{N}{\tau_e}=0$

$P \Rightarrow (I-I_{th})$ which is expressed as $P=\frac{\tau_p}{e}(I-I_{th})$,

with $I_{th}=\frac{e}{\tau_e}(N_0+\frac{1}{A\tau_p})$.

$\longrightarrow$

1.2 cont

P is valid above the threshold. Above the threshold Current there is stimulated emission and the Photon increase with bias current according to a slope $\frac{I\tau_p}{e}$.

$$I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$I_{th} = 14.4\,mA$.

1.3

The carrier number will be clamped at $N = \left(N_0 + \frac{1}{A\tau_p}\right)$ above threshold, even if the current increases the Carrier number will stays at this value due to the gain = losses.

But below the threshold, The carrier number increases with the increase of the current. the increase will be according to this expression.

$$N = \frac{\tau_e}{e} I.$$

Q2:-

2.1:-

since it is in the steady state, we get rid off the spontaneous emission $\beta \frac{N}{\tau_e}$.

$$\Rightarrow \begin{cases} \frac{d(\bar{N}+dN)}{dt} = \frac{\bar{I}+dI}{e} - A(\bar{N}+dN-N_0)(\bar{P}+dP) - \frac{\bar{N}+dN}{\tau_e} \\ \frac{d(\bar{P}+dP)}{dt} = A(\bar{N}+dN-N_0)(\bar{P}+dP) - \frac{(\bar{P}+dP)}{\tau_p} \end{cases}$$

we set dN and dP to zero.

The equations for the perturbation is as follow:-

$$\Rightarrow \begin{cases} \frac{d(dN)}{dt} = \frac{dI}{e} - A(\bar{N}-N_0)dP - (A\bar{P}+\frac{1}{\tau_e})dN \\ \frac{d(dP)}{dT} = A((\bar{N}-N_0) - \frac{1}{\tau_p})dP + A\bar{P}dN \end{cases}$$

This analysis is carried out above threshold, so:-

$$\begin{cases} \frac{d(dN)}{dt} = \frac{dI}{e} - \frac{dP}{\tau_p} - (A\bar{P}+\frac{1}{\tau_e})dN \\ \frac{d(dP}{dt} = A\bar{P}dN \end{cases}$$

next change those equation to fit the matrical form:-

$$\begin{pmatrix} \frac{d(dN)}{dt} \\ \frac{d(dP)}{dt} \end{pmatrix} = \begin{pmatrix} -(A\bar{P}+\frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} dN \\ dP \end{pmatrix} + \begin{pmatrix} dI/e \\ 0 \end{pmatrix}$$

Q2:-

2.2

$$M = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix}$$

τ_p = ~~carrier~~ Photon lifetime.

τ_e = carrier lifetime

A = The gain coefficient.

$\bar{P}$ = the first order perturbation of the photon number.

2.3

In the fourier domain:-

$$\Rightarrow \begin{pmatrix} j\omega\,\widetilde{\delta N} \\ j\omega\,\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix} \text{ with } M = \begin{pmatrix} j\omega + (A\bar{P} + \frac{1}{\tau_e}) & \frac{1}{\tau_p} \\ -A\bar{P} & j\omega \end{pmatrix}$$

and $\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1} \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix} \Rightarrow M^{-1} = \frac{1}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\bar{P} - j\omega & -(A\bar{P} + \frac{1}{\tau_e}) \end{pmatrix}$

$\omega_R^2 = \frac{A\bar{P}}{\tau_p}$ and $\frac{2}{\tau_R} = (A\bar{P} + \frac{1}{\tau_e})$. ω_R is the relaxation oscilation and $\frac{2}{\tau_R}$ is the dumping coefficient.

$$\Rightarrow \widehat{\delta P} = \frac{-A\bar{P}}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \widehat{\delta I}/e.$$

$$\Rightarrow \frac{\widehat{\delta P}}{\widehat{\delta I}} = \frac{-\tau_p \omega_R^2 / e}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \qquad \Rightarrow \widetilde{\delta N} = \frac{-j\omega}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \widehat{\delta I}/e$$

Q3:

3.1

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right) P_s + \frac{4KT}{R_c}\right)\Delta f}$$

$\eta = 0.85$, $\lambda = 1550$ nm, $\Delta F = 20GHz$, $R_c = 50\ \Omega$,

$T = 300$ k, $P_s = -3$ dBm $\Rightarrow P_s = 0.501$ mW $\Rightarrow 0.501 \times 10^{-3}$

$h\nu = hc/\lambda$, so we can express the equation as follows:

$$\frac{S}{N} = \frac{\left(\frac{0.85e}{hc/\lambda}\right)^2 P_s^2}{\left(2e\left(\frac{0.85e}{hc/\lambda}\right) P_s + \frac{4KT}{R_c}\right)\Delta F} . \quad 0.85e = 1.36 \times 10^{-19}$$

$$\Rightarrow \frac{hc}{\lambda} \Rightarrow \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1550 \times 10^{-9}} = \frac{1.986 \times 10^{-25}}{1550 \times 10^{-9}} = 1.28 \times 10^{-19}$$

$$\frac{S}{N} = \frac{\left(\frac{0.85e}{1.28 \times 10^{-19}}\right)^2 \times (0.5\cancel{1} \times 10^{-3})^2}{\left(2e(1.062)(0.501 \times 10^{-3}) + \frac{(4)(1.38 \times 10^{-23})(300)}{50}\right) 20 \times 10^{9}}$$

$$\frac{S}{N} = 44 \text{ dB}.$$

3.2

if the temperature is $0^\circ C$ that mean $k = 273$ k.

So: $\frac{S}{N} \approx 44.4$ dB

Q 3:-

3.3

if the temperature increases, the thermal noise increases. The noise level increases, thus, S/N decreases.

3.4

when Δf increases, this mean that the Photodiode can capture more noise, which in turn increase the noise level and the S/N decreases.

Q4:

4.1

$\tau_{ne} = 75$ ns , $\tau_e = 50$ ns, $\lambda = 450$ nm, $I = 300$ mA,
$n = 3.6$

The internal quantum efficiency $= \left[1 + \frac{\tau_e}{\tau_{ne}}\right]^{-1}$

$$\Rightarrow \left[1 + \frac{50}{75}\right]^{-1} \Rightarrow [1.66]^{-1}$$

$$\Rightarrow 0.602.$$

The extraction efficiency is expressed as:

$$\eta = \eta_{int} \times K \times \left[\frac{1}{(n+1)^2}\right] \times \frac{1}{n}$$

$$\Rightarrow \eta = (0.602) \times (K) \times \left[\frac{1}{(3.6+1)^2}\right] \times \frac{1}{3.6}$$

$$\Rightarrow \eta = 0.602K \times [0.0472] \times 0.277$$

$$\Rightarrow \eta = 0.00787K$$, K is a constant.