



Coursework Submission Cover Sheet

Student No	13210818	Degree Scheme	22.00%
Student Name	Sari Al Gaidy	Year	2015
Module	EE559	Lecturer	Dr. Pascal Landais
Title	Semester two solution assignment 2015	Hours spent on this exercise	25 hours

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Signed:

Date:

24/04/2015

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Comments:

Q1:

1.1:

They describe the time evolution of the carrier and photonic numbers.

P = The Photon number.

N = The carrier number.

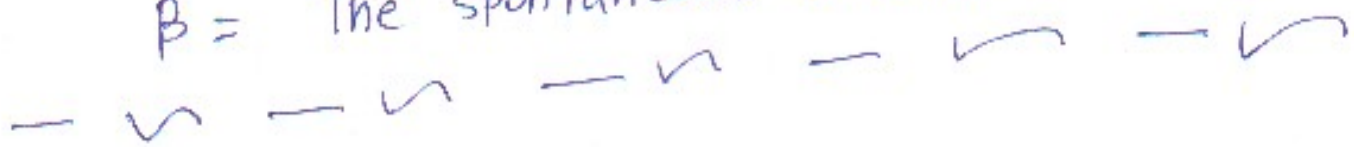
A = The gain coefficient.

I = The value of the bias current applied to the laser.

τ_c = The carrier lifetime.

τ_p = The photon lifetime.

β = The spontaneous emission coupling.



1.2

To get the threshold current, the rate equation must be used in steady state, which means there is no time variation. $\frac{d}{dt} = 0$. At threshold current, the spontaneous emission is excluded. ~~and gain~~
Gain = losses.

$A(N - N_0) = \frac{1}{\tau_p}$. carrier number at threshold = $(N_0 + \frac{1}{A\tau_p})$,

which is used in the equation: $\frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_c} = 0$

$P \Rightarrow (I - I_{th})$ which is expressed as $P = \frac{I - I_{th}}{e}$,

with $I_{th} = \frac{e}{\tau_c} (N_0 + \frac{1}{A\tau_p})$.

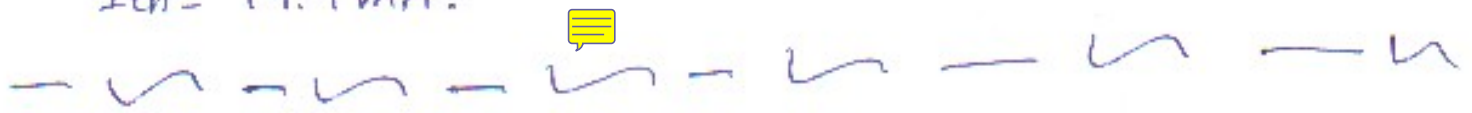


1.2 cont

P is valid above the threshold. Above the threshold current there is stimulated emission and the photon increase with bias current according to a slope $\frac{\tau_p}{e}$.

$$I_{th} = \frac{e}{\tau_e} \left(N_0 + \frac{1}{A\tau_p} \right)$$

$$I_{th} = 14.4 \text{ mA.}$$



1.3

The carrier number will be clamped at $N = \left(N_0 + \frac{1}{A\tau_p} \right)$ above threshold, even if the current increases the carrier number will stay at this value due to the gain = losses.

But below the threshold, the carrier number increases with the increase of the current. the increase will be according to this expression.

$$N = \frac{\tau_e}{e} I.$$

Q2:

2.1:-

since it is in the steady state, we get rid of the spontaneous emission $\beta \frac{N}{\tau_e}$.

$$\Rightarrow \begin{cases} \frac{d(\bar{N} + \delta N)}{dt} = \frac{\bar{I} + \delta I}{e} - A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{N} + \delta N}{\tau_e} \\ \frac{d(\bar{P} + \delta P)}{dt} = A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{(\bar{P} + \delta P)}{\tau_p} \end{cases}$$

we set δN and δP to zero.

The equations for the perturbation is as follow:-

$$\Rightarrow \begin{cases} \frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(\bar{N} - N_0)\delta P - (A\bar{P} + \frac{1}{\tau_e})\delta N \\ \frac{d(\delta P)}{dt} = A((\bar{N} - N_0) - \frac{1}{\tau_p})\delta P + A\bar{P}\delta N \end{cases}$$

This analysis is carried out above threshold, so:-

$$\begin{cases} \frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \frac{\delta P}{\tau_p} - (A\bar{P} + \frac{1}{\tau_e})\delta N \\ \frac{d(\delta P)}{dt} = A\bar{P}\delta N \end{cases}$$

next change those equation to fit the matricial form:-

$$\begin{pmatrix} \frac{d(\delta N)}{dt} \\ \frac{d(\delta P)}{dt} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

Q2:-

2.2

$$M = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix}$$

τ_p = ~~carrier~~ photon lifetime.

τ_e = carrier lifetime

A = The gain coefficient.

\bar{P} = the first order perturbation of the photon number.

2.3

In the fourier domain:-

$$\Rightarrow \begin{pmatrix} j\omega \hat{N} \\ j\omega \hat{P} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \hat{N} \\ \hat{P} \end{pmatrix} + \begin{pmatrix} \hat{I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} \hat{N} \\ \hat{P} \end{pmatrix} = \begin{pmatrix} \hat{I}/e \\ 0 \end{pmatrix} \text{ with } M = \begin{pmatrix} j\omega + (A\bar{P} + \frac{1}{\tau_e}) & \frac{1}{\tau_p} \\ -A\bar{P} & j\omega \end{pmatrix}$$

$$\text{and } \begin{pmatrix} \hat{N} \\ \hat{P} \end{pmatrix} = M^{-1} \begin{pmatrix} \hat{I}/e \\ 0 \end{pmatrix} \Rightarrow M^{-1} = \frac{1}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\bar{P} & -j\omega - (A\bar{P} + \frac{1}{\tau_e}) \end{pmatrix}$$

$\omega_R^2 = \frac{A\bar{P}}{\tau_p}$ and $\frac{2}{\tau_R} = (A\bar{P} + \frac{1}{\tau_e})$. ω_R is the relaxation oscillation and $\frac{2}{\tau_R}$ is the dumping coefficient.

$$\Rightarrow \hat{P} = \frac{-A\bar{P}}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \hat{I}/e.$$

$$\Rightarrow \frac{\hat{P}}{\hat{I}} = \frac{-\tau_p \omega_R^2}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \Rightarrow \hat{N} = \frac{-j\omega}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \hat{I}/e$$

Q 3:

3.1

$$\frac{S}{N} = \frac{\left(\frac{ne}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{ne}{h\nu}\right) P_s + \frac{4kT}{R_c}\right) \Delta F}$$

$$n=0.85, \lambda=1550 \text{ nm}, \Delta F=20 \text{ GHz}, R_c=50 \Omega,$$

$$T=300 \text{ K}, P_s = -3 \text{ dBm} \Rightarrow P_s = 0.501 \text{ mW} \Rightarrow 0.501 \times 10^{-3}$$

$h\nu = hc/\lambda$, so we can express the equation as follows:

$$\frac{S}{N} = \frac{\left(\frac{0.85e}{hc/\lambda}\right)^2 P_s^2}{\left(2e\left(\frac{0.85e}{hc/\lambda}\right) P_s + \frac{4kT}{R_c}\right) \Delta F} \cdot 0.85e = 1.36 \times 10^{-19}$$

$$\Rightarrow \frac{hc}{\lambda} \Rightarrow \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1550 \times 10^{-9}} = \frac{1.986 \times 10^{-25}}{1550 \times 10^{-9}} = 1.28 \times 10^{-19}$$

$$\frac{S}{N} = \frac{\left(\frac{0.85e}{1.28 \times 10^{-19}}\right)^2 \times (0.501 \times 10^{-3})^2}{\left(2e(1.062)(0.501 \times 10^{-3}) + \frac{(4)(1.38 \times 10^{-23})(300)}{50}\right) 20 \times 10^9}$$

$$\frac{S}{N} = 44 \text{ dB.}$$

3.2

if the temperature is 0°C that mean $k = 273 \text{ K}$.

$$\text{So: } \frac{S}{N} \approx 44.4 \text{ dB}$$

Q 3:-

3.3

if the temperature increases, the thermal noise increases. The noise level increases, thus, S/N decreases.

3.4

when Δf increases, this means that the photodiode can capture more noise, which in turn increases the noise level and the S/N decreases.

Q4:

4.1

$$\tau_{ne} = 75 \text{ ns}, \tau_e = 50 \text{ ns}, \lambda = 450 \text{ nm}, I = 300 \text{ mA}, \\ n = 3.6$$

$$\text{The internal quantum efficiency} = \left[1 + \frac{\tau_e}{\tau_{ne}} \right]^{-1}$$

$$\Rightarrow \left[1 + \frac{50}{75} \right]^{-1} \Rightarrow [1.66]^{-1}$$

$$\Rightarrow 0.602.$$

The extraction efficiency is expressed as:

$$\eta = \eta_{int} \times K \times \left[\frac{1}{(n+1)^2} \right] \times \frac{1}{m}$$

$$\Rightarrow \eta = (0.602) \times (K) \times \left[\frac{1}{(3.6+1)^2} \right] \times \frac{1}{3.6}$$

$$\Rightarrow \eta = 0.602K \times [0.0472] \times 0.277$$

$$\Rightarrow \eta = 0.00787K, \quad K \text{ is a constant.}$$

