Brian Murray, EE506 Assignment 2015

Q1: 1.1 These equations describe the evolution of the carrier number and photon number, of a laser, over time.
The symbols used are identified below:

N = Carrier number
t = time
I = applied bias current
e = |charge on electron|
A = gain coefficient.
N_0 = carrier number at transparency (when spontaneous recombination = stimulated generation)
P = photon number
τ_e = carrier lifetime
τ_p = photon lifetime
β = spontaneous emission coupling fraction

1.2 Rate equations:
$$\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e} \quad (1)$$
$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e} \quad (2)$$

- Steady-state $\Rightarrow \frac{dN}{dt} = \frac{dP}{dt} = 0$
- Assumption: Stimulated emission >> spontaneous emission at treshold
  $\hookrightarrow A(N-N_0)P \gg \beta\frac{N}{\tau_e}$, so let $\beta\frac{N}{\tau_e} \to 0$

Then, using (2) we get: $A(N-N_0)P = \frac{P}{\tau_p} \Rightarrow N = N_0 + \frac{1}{A\tau_p}$
Fill this into (1):
$$I/e - A(N-N_0)P - \frac{N}{\tau_e} = 0$$
$$\Rightarrow I/e - A\left(N_0 + \frac{1}{A\tau_p} - N_0\right)P - \frac{N_0 + \frac{1}{A\tau_p}}{\tau_e} = 0$$
$$\Rightarrow I = \frac{e}{\tau_p}P + \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right) \text{ at threshold.}$$

But, $P > 0$ only after treshold is reached, and so set $P=0$:

$$\therefore \boxed{I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)}$$

1.3: Above threshold, the carrier number is clamped to $N = N_0 + \frac{1}{A\tau_p}$ as the gain equals the losses. Any excess carriers are used directly for stimulated emission.

Below treshold, the carrier number increases linearly with current, according to $N = \left(\tau_e/e\right) I$.

Q2: 2.i Rate eqns:

$$\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e}$$
$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e}$$

Neglect spontaneous emission, so $\beta\frac{N}{\tau_e} \to 0$.
Apply a small perturbation:

$$\frac{d(\bar{N}+\delta N)}{dt} = \frac{\bar{I}+\delta I}{e} - A(\bar{N}+\delta N - N_0)(\bar{P}+\delta P) - \frac{\bar{N}+\delta N}{\tau_e}$$
$$\frac{d(\bar{P}+\delta P)}{dt} = A(\bar{N}+\delta N - N_0)(\bar{P}+\delta P) - \left(\frac{\bar{P}+\delta P}{\tau_p}\right)$$

After multiplying this out, we set all second order terms to zero i.e. $\delta N \delta P \to 0$.
Also we note that $\frac{d\bar{N}}{dt}$ and $\frac{d\bar{P}}{dt} = 0$, still.

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(\bar{N}-N_0)\delta P - \left(A\bar{P} + \frac{1}{\tau_e}\right)\delta N$$
$$\frac{d(\delta P)}{dt} = \left[A(\bar{N}-N_0) - \frac{1}{\tau_p}\right]\delta P + A\bar{P}\delta N$$

Now recall $A(\bar{N}-N_0) = \frac{1}{\tau_p}$ as we are above threshold and so the gain = losses.

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \frac{\delta P}{\tau_p} - \left(A\bar{P} + \frac{1}{\tau_e}\right)\delta N$$
$$\frac{d(\delta P)}{dt} = A\bar{P}\delta N$$

Then writing this in matrix form:

$$\boxed{\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \underbrace{\begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -1/\tau_p \\ A\bar{P} & 0 \end{pmatrix}}_{=M} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}}$$

2.2: M describes the rate of change of the perturbation in the carrier and photon numbers of the laser due to a perturbation in current.

A = Gain constant

$\bar{P}$ = photon number

τ_e = carrier lifetime

τ_p = photon lifetime

~~2.3: In the frequency domain (after FT), the matrix elements in 2.2 are invariant~~

2.3: The elements of M are invariant under Fourier transformation.

Apply FT:

$$\begin{pmatrix} i\omega\,\widetilde{\delta N} \\ i\omega\,\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}$$

A little factorisation gives:

$$\underbrace{\begin{pmatrix} i\omega + (A\bar{P} + \frac{1}{\tau_e}) & \frac{1}{\tau_p} \\ -A\bar{P} & i\omega \end{pmatrix}} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}$$

Inverse of this: $$\frac{1}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_e})} \begin{pmatrix} -i\omega & \frac{1}{\tau_p} \\ -AP & -i\omega - (A\bar{P} + \frac{1}{\tau_e}) \end{pmatrix}$$

Then we see our answer:

$$\boxed{\begin{aligned} \widetilde{\delta P} &= \frac{-A\bar{P}}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_e})} \frac{\widetilde{\delta I}}{e} \\ \widetilde{\delta N} &= \frac{-i\omega}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_e})} \frac{\widetilde{\delta I}}{e} \end{aligned}}$$

Q3: 3.1:
$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_c}\right)\Delta f}$$

$\eta = .85$

$\lambda_{max\,power} = 1550\ nm \Rightarrow \nu = \frac{c}{\lambda} = 1.935 \times 10^{14}\ Hz$

$\Delta f = 20\ GHz$

$R_c = 50\ \Omega$

$T = 300\ K$

$P = -3\,dBm = \frac{10^{\left(\frac{-3}{10}\right)}}{1000}$ Watts $= .0005$ Watts

Input into Wolfram Mathematica!

Result: $\frac{S}{N} = 28218.9$

$\Rightarrow S/N = 44.5\ dB$

3.2: Now $T = 0°C = 273\ K$

Put it back into Mathematica:

Result: $S/N = 30002.5$

$\Rightarrow S/N = 44.8\ dB$

Brian Murray, EE506 Assignment 2015, Question 3

Setting the parameters:

$h = 6.62\times 10^{-34};$
$\nu = 0.85;$
$P_{dBm} = -3;$
$T_1 = 300;$
$R_c = 50;$
$\Delta_f = 20\times 10^{9};$
$c = 3\times 10^{8};$
$e = 1.6\times 10^{-19};$
$k = 1.38\times 10^{-23};$
$\lambda = 1550\times 10^{-9};$

Formula for converting power in watts to power in dBm:

$P_W = 10^{(P_{dBm}/10)} / 1000;$

- **Question 3.1**

Calculating the signal-to-noise ratio in a linear scale:

$$N\left[\frac{\left(\frac{\nu e}{h c/\lambda}\right)^2 (P_W)^2}{\left(2 e \left(\frac{\nu e}{h c/\lambda}\right)(P_W) + \frac{4 k T_1}{R_c}\right)(\Delta_f)}\right]$$

28 218.9

Calculating the signal-to-noise ratio in dB:

$P_{dB} = 10\,\mathrm{Log10}[\%]$

44.5054

- **Question 3.2:**

Setting the new temperature in kelvin:

$T_2 = 273.15;$

Calculating the signal-to-noise ratio in a linear scale:

$$N\left[\frac{\left(\frac{\nu e}{h c/\lambda}\right)^2 (P_W)^2}{\left(2 e \left(\frac{\nu e}{h c/\lambda}\right)(P_W) + \frac{4 k T_2}{R_c}\right)(\Delta_f)}\right]$$

29 991.9

Calculating the signal-to-noise ratio in dB:

$$P_{dB} = 10\,\text{Log10}[\%]$$

44.77

Q4:

4.1: Internal quantum efficiency: $\eta_{int} = \dfrac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}}$

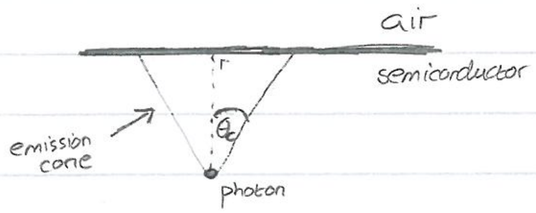
$$\Rightarrow \eta_{int} = \frac{50e9}{50e9 + 75e9}$$

$$= \frac{50}{50+75}$$

$$\Rightarrow \eta_{int} = \frac{2}{5} = .4$$

~~[illegible]~~

Extraction efficiency: There are two factors we need to take into account:



θ_c = critical angle

First, we find the number of photons that exit through this ~~escape~~ emission cone (i.e. those that are not totally internally reflected.)

This will be the ratio of the solid angle subtended by the photon to the solid angle through which all the photons can travel (i.e., a full sphere.)

$$\therefore \eta_{extr1} = \frac{2\pi\int_0^{\theta_c}\sin\theta\, d\theta}{4\pi} = \frac{1}{2}(1-\cos\theta_c)$$

Now use $\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{sc}}\right) = \sin^{-1}\left(\frac{1}{3.6}\right) = .281 \text{ rad} = 16.13°$

$$\therefore \eta_{extr1} = \frac{1}{2}(1-\cos(16.13°)) = .0197$$

The second factor involves the partial reflection of the photons' energy at the sc-air interface. (Treat photon as a wave and use transmission and reflection coefficients).

PTO

The reflectance of the facet is given by

$$T = \frac{n_{air}\cos\theta_i}{n_{sc}\cos\theta_i} t^2$$ where θ_i = photon incident angle

and $$t = \frac{2 n_{sc}\cos\theta_i}{n_{air}\cos\theta_i + n_{sc}\cos\theta_i}$$

Thus, $$t = \frac{2(3.6)}{1 + 3.6} = 1.565$$

and then,

$$T = \frac{1}{3.6}(1.565)^2 = .68$$

So, finally, the extraction efficiency is

$$\eta_{extraction} = \eta_{extr1} . T = (.0197)(.68) = .013$$