Appendix A Student Declaration of Academic Integrity

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Declaration

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Programme: MMEng.

Module Code: ES06

Assignment Title: EE 506_Assignment_2015.pdf.

Submission Date: 24 - April - 2015

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

I have identified and included the source of all facts, ideas, opinions, viewpoints of others in the assignment references. Direct quotations from books, journal articles, internet sources, module text, or any other source whatsoever are acknowledged and the sources cited are identified in the assignment references.

I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

I have read and understood the DCU library referencing guidelines (available at: http://www.library.dcu.ie/LibraryGuides/DCU Library Guide to Harvard Style of Citing & Referencing/player.html Referencing/player.html accessed 9th January 2013) and/or recommended in the assignment guidelines and/or programme documentation.

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Name: Clifford Def

Date: 24 - April - 2015

Report of Suspected Plagiarism / Breach of Academic Integrity

Questions 1

1.1 [5 marks]

1. The set of equations given below are called rate equations:

$$\begin{cases} \frac{dN}{dt} = \frac{I}{e} - A(N - No)P - \frac{N}{\tau_e} \\ \frac{dP}{dt} = A(N - No)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e} \end{cases} \quad (1)$$

What does this set of equations describe? Identify and name all the symbols appearing in Eq. 1. This set of equations describes the time evolution of both the carrier number and the photon number.

Answer:

The Rate equation describes the how a laser diode performs; it is a model of the lasers electrical and optical performance. The two differentiation equations describe the time evolution of the carrier and photon numbers.

The differential equation below is the time variation of the charged carriers (N).	The differential equation below is the time variation of the Photons (P)
$\frac{dN}{dt} = \frac{I}{e} - A(N - No)P - \frac{N}{\tau_e}$	$\frac{dP}{dt} = A(N - No)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e}$
I is the value of the bias current applied to the laser	β is the spontaneous emission coupling
e is the electron charge	Tp is the lifetime of the photon
$\frac{I}{e}$ is the injected carriers into the laser	
A= gain coefficient (the slope of linear independence)	
Te is the carrier lifetime	
P is the Photon Density	
N is the Carrier Density	
No is the carrier number at transparency	

1.2	[15 marks]
Use Eq. (1) to determine the expression for the current threshold and from the list of constants attached to this exam paper calculate the threshold current. What assumption do you use to facilitate this calculation?	

Answer

The rate equation can be used to determine the current threshold at steady state.

At this condition the steady state is equal to $\frac{dN}{dt} = 0$ and $\frac{dP}{dt} = 0$ thus $\frac{d}{dt} = 0$

Step 1: For the Photon differentiation equation, assume that spontaneous emission can be ignored because it is relatively zero. $\frac{dP}{dt} = 0$	$\frac{dP}{dt} = A(N - N_o)P - \frac{P}{Tp} + \beta\frac{N}{Te}$ $B\frac{N}{Te} \approx 0$ thus Formula simplified to $0 = A(N-N_o)P - \frac{P}{Tp}$
Step 2: The steady state the gain is equal to the losses Divide by P (To cancel P on both sides) Divide both sides by A (they cancel) Moving 'N_o' across	$A(N-N_o)P = \frac{P}{Tp}$ $A(N-N_o) = \frac{1}{Tp}$ $(N-N_o) = \frac{1}{ATp}$ $N = \frac{1}{ATp} + N_o$ (This expression can now be substituted later on into the carrier equation)

Step 3:	
Carrier Equation	$\frac{dN}{dt} = \frac{I}{e} - A(N - N_o)P - \frac{N}{Te}$
$\frac{dN}{dt} = 0$	$0 = \frac{I}{e} - A(N-N_o)P - \frac{N}{Te}$ Same as
Moving a $A(N - N_o)P$ across	$A(N - N_o)P = \frac{I}{e} - \frac{N}{Te}$
From equation in step 2, substitute N on both sides	
$N = \frac{1}{A.Tp} + N_o$	$A((\frac{1}{A.Tp} + No) - No)P = \frac{I}{e} - \frac{\frac{1}{A.Tp}+No}{Te}$
Now simplify ('No' & A cancel, bring P up and multiply by Tp	$\frac{P}{Tp} = \frac{I}{e} - \frac{1}{Te}(\frac{1}{A.Tp} + No)$
In terms of P, (multiply across by Tp)	$P = \frac{I.Tp}{e} - \frac{Tp}{Te}(\frac{1}{A.Tp} + No)$
Multiply the brackets	$P = \frac{I.Tp}{e} - \frac{1}{A.Te.} - \frac{Tp.No}{Te}$
Factor out $\frac{Tp}{e}$	$P = \frac{Tp}{e}(I - \frac{e}{A.Tp.Te} - \frac{No.\ e}{Te})$

<table>
<tr>
<td>Step 4
P expressed as (I - I_{th})

Thus, the Threshold current is equal to</td>
<td>

$P = \frac{Tp}{e}(I - I_{th})$

P is always positive and is valid only above the threshold. Above threshold current, there is stimulated emission, and the photon number increases with bias current according to a slope $\frac{Tp}{e}$

So $Ith = (\frac{e}{A.Tp.Te} - \frac{No.\ e}{Te})$ thus

$I_{th} = \frac{e}{Te}(No + \frac{1}{A\,Tp})$

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<tr>
<td>Using the constants to work out the Threshold current (Is)</td>
<td>
<table>
<tr><td>e</td><td>$1.6 \times 10^{-19} C$</td></tr>
<tr><td>c</td><td>$3 \times 10^{8} m/s$</td></tr>
<tr><td>h</td><td>$6.62 \times 10^{-34} Js$</td></tr>
<tr><td>k</td><td>$1.38 \times 10^{-23} J/K$</td></tr>
<tr><td>cavity length</td><td>$250 \mu m$</td></tr>
<tr><td>active region width</td><td>$2 \mu m$</td></tr>
<tr><td>active region thickness</td><td>$200 nm$</td></tr>
<tr><td>N_o</td><td>1.2×10^{8}</td></tr>
<tr><td>A</td><td>$8 \times 10^{3} s^{-1}$</td></tr>
<tr><td>τ_p</td><td>$1.6 ps$</td></tr>
<tr><td>τ_e</td><td>$2.2 ns$</td></tr>
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<td>Working out the formula</td>
<td>

=> $I_{th} = \frac{e}{Te}(No + \frac{1}{A\,Tp})$

=> $\frac{1.6 X 10-19}{2.2 X10-9}$ (1.2 x 10^{8} + $\frac{1}{(8 X 10+3)(1.6 X10-12)}$)

=> (7.272 x 10^{-11})(1.2 x 10^{8} + 78.125 x 10^{6})
=> 7.272 x 10^{-11} (198.125 x 10^{6})

=> 14.407 mA = (Value of I_{th})

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Assumptions	An assumption is this calculation was the fact that at the threshold current the spontaneous emission can be neglected, thus $B\frac{N}{Te} \approx 0$ and the gain is equal to the losses.

Q 1.3

Determine the expression for the number of carriers as a function of the bias current set above threshold and below the threshold. Comment these results.

$\frac{dN}{dt} = \frac{I}{e} - GP - \frac{N}{Te}$ $\frac{dP}{dt} = GP + \beta\frac{N}{Te} - \frac{P}{Tp}$

Above Threshold

GP >> $\beta\frac{N}{T}$

- P increases with Current ($P = \frac{Tp}{e}(I - I_{th})$)
- Above threshold, the carrier number is clamped to ($No + \frac{1}{A.Tp}$).

Even though, the bias current can increase the carrier number will stay at this value. I.E Independent of the bias current, the value of N is fixed

Below Threshold

GP << $\beta\frac{N}{T}$

- When the bias current is set below the threshold, the carrier number will increase with the bias current.
- Therefore more and more carriers are injected into the junction according to the linear expression for the current.
- So dN/dt=0 and P is negligible so
- $\frac{dN}{dt} = \frac{I}{e} - A(N - No)P - \frac{N}{Te}$ => $0 = \frac{I}{e} - A(N - No)0 - \frac{N}{Te}$

Therefore $\frac{I}{e} = \frac{N}{Te}$ => To find Carriers **N =** $\frac{(Te)(I)}{e}$

Question 2

Q2.1

If $\bar{N}$, $\bar{P}$ and $\bar{I}$ represent steady state quantities, carry out a small signal analysis for the rate equations (1) from Question 1, for the following perturbations δN, δP and δI around these steady state points. Your solution must be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$

where M is a 2×2 matrix.

Given Formula from Question 1

$$\frac{dN}{dt} = \frac{I}{e} - A(N - No)\,P - \frac{N}{Te}$$

$$\frac{dP}{dt} = A(N - No)\,P - \frac{P}{Tp} - \beta\,\frac{N}{Te}$$

Step 1: • Add in δN δ P and δI • Spontaneous emissions can be neglected $\beta\,\frac{N}{Te}$	$\frac{d(\bar{N} + \delta N)}{dt} = \frac{\bar{I} + \delta I}{e} - A(N+\delta N - No)\,(P+\delta P) - \frac{N + \delta N}{Te}$ $\frac{d(P + \delta P)}{dt} = A(N + \delta N) - No)\,(P + \delta P) - \frac{(P + \delta P)}{Tp}$
Step 2: • Analysis will be limited to first order, δN and δP =0	$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(N - No)\,\delta P - (AP + \frac{1}{Te})\,\delta N$ $\frac{d(\delta P)}{dt} = (A(N - No) - \frac{1}{Tp})\,(\delta P) + AP\delta N$
Step 3: • This scenario is above-threshold so $A(N - No) = \frac{1}{Tp}$	$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \frac{\delta P}{Tp} - (AP + \frac{1}{Te})\,\delta N$

	$\frac{d(\delta P)}{dt} = AP\delta N$
<u>Step 4:</u> • Writing equations as a matrix Form	$\begin{pmatrix} \frac{d(\delta N)}{dt} \\ \frac{d(\delta P)}{dt} \end{pmatrix} = \begin{pmatrix} -\left(AP + \frac{1}{Te}\right) - & \frac{1}{Tp} \\ AP & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$

Q2.2

Identify all the elements of the matrix, M.

The elements of matrix

$$M = \begin{pmatrix} -(A\bar{P} + \frac{1}{Te} & -\frac{1}{Tp} \\ AP & 0 \end{pmatrix}$$

Q2.3

If $\widetilde{\delta N}$, $\widetilde{\delta P}$ and $\widetilde{\delta I}$ are the expression of the first order perturbation of δN, δP and δI in the Fourier domain, derive the expressions for $\widetilde{\delta N}$ and $\widetilde{\delta P}$ as a function $\widetilde{\delta I}$.

Hint:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{(ad - cb)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad (3)$$

Step 1: Fourier transforms substitute

Fourier Transformation will require substitute of the following

- $\delta N(t) \rightarrow \delta N(w)$
- $\delta N(t) \rightarrow j\omega\delta N(w)$
- $\delta I(t) \rightarrow \ \delta I\ (w)$,
- $\delta P(t) \rightarrow \delta P\ (w)$
- $\delta P(t) \rightarrow j\omega\delta P\ (w)$

$$\begin{pmatrix}\frac{d(\delta N)}{dt}\\ \frac{d(\delta P)}{dt}\end{pmatrix} = \begin{pmatrix} -\left(A\bar{P}+\frac{1}{Te}\right) & -\frac{1}{Tp} \\ A\bar{P} & 0\end{pmatrix}\begin{pmatrix}\delta N\\ \delta P\end{pmatrix}\begin{pmatrix}\delta I/e\\ 0\end{pmatrix}$$

Substitute in the following into $\frac{d(\delta N)}{dt} \rightarrow j\omega\delta N(w)$ *and* $\frac{d(\delta P)}{dt} \rightarrow j\omega\delta P\ (w)$	$\begin{bmatrix} j\omega\delta N \\ j\omega\delta P\end{bmatrix} = \begin{bmatrix} -\left(A\bar{P}-\frac{1}{\tau_e}\right) & -\frac{1}{\tau_P} \\ A\bar{P} & 0\end{bmatrix}\begin{bmatrix}\widetilde{\delta N} \\ \widetilde{\delta P}\end{bmatrix} + \begin{bmatrix}\widetilde{\delta I}/e \\ 0\end{bmatrix}$
This can be written as	$M\begin{pmatrix}\widetilde{\delta N} \\ \widetilde{\delta P}\end{pmatrix} = \begin{bmatrix}\widetilde{\delta I}/e \\ 0\end{bmatrix}$
And M is equal to	$\begin{bmatrix} j\omega + \left(A\bar{P}+\frac{1}{\tau_e}\right) + \frac{1}{\tau_P} \\ -A\bar{P} & j\omega\end{bmatrix}$ => $\begin{bmatrix}\widetilde{\delta N} \\ \widetilde{\delta P}\end{bmatrix} = M^{-1} + \begin{bmatrix}\widetilde{\delta I}/e \\ 0\end{bmatrix}$
Inverse matrix M^{-1} $\begin{bmatrix} j\omega + A\bar{P} + \frac{1}{\tau_e} & \frac{1}{\tau_P} \\ -A\bar{P} & j\omega\end{bmatrix}^{-1} = \frac{1}{j\omega\left(j\omega + A\bar{P} + \frac{1}{\tau_e}\right) + \frac{A\bar{P}}{\tau_P}}\begin{bmatrix} j\omega & -\frac{1}{\tau_P} \\ A\bar{P} & j\omega + A\bar{P} + \frac{1}{\tau_e}\end{bmatrix}$	

$$\begin{bmatrix} j\omega + \ A\bar{P} + \frac{1}{\tau_e} & \frac{1}{\tau_P} \\ -A\bar{P} & j\omega \end{bmatrix}^{-1} = \frac{1}{-\ \omega^2 + \ j\omega A\bar{P} + \frac{j\omega}{\tau_e} + \frac{A\bar{P}}{\tau_P}} \begin{bmatrix} j\omega & -\frac{1}{\tau_P} \\ A\underline{P} & j\omega + \ A\bar{P} + \frac{1}{\tau_e} \end{bmatrix}$$

$$= \quad \frac{1}{w^2 - \ j\omega\left(A\bar{P} + \frac{1}{\tau_e}\right) - \frac{A\bar{P}}{\tau_P}} \begin{bmatrix} -j\omega & \frac{1}{\tau_P} \\ -A\bar{P} & -j\omega - \ A\bar{P} + \frac{1}{\tau_e} \end{bmatrix}$$

$$M^{-1} = \frac{1}{\omega 2 - \frac{A\bar{P}}{\tau_P} - \ j\omega\left(\ A\bar{P} + \frac{1}{\tau_e}\right)} \begin{bmatrix} -j\omega & \frac{1}{\tau_P} \\ -A\bar{P} & -\ j\omega - (\ A\bar{P} + \frac{1}{\tau_e}) \end{bmatrix}$$

We now can substitute the following

- $w^2{}_r = \frac{AP}{\tau_P}$
- $\frac{2}{\tau_r}$ = (AP $+ \frac{1}{\tau_e}$)ω_r is the relaxation oscillation
- $\frac{2}{\tau_r}$ is the dumping coefficient

Thus:

$$\widetilde{\delta P} = \frac{-A\bar{P}}{\omega^2 - \frac{A\bar{P}}{\tau_P} - j\omega(A\bar{P} + \frac{1}{\tau_e})} \quad \frac{\tilde{\delta}\tilde{I}}{e},$$

	$$\frac{\widetilde{\delta P}}{\delta \tilde{I}} = \frac{\frac{-\tau_P \omega^2 r}{e}}{\omega^2 - \frac{-A\bar{P}}{\tau_P} - J\omega(A\bar{P} + \frac{1}{\tau_e})}$$ $$\widetilde{\delta N} = \frac{-j\omega}{\omega^2 - \frac{-A\bar{P}}{\tau_P} - J\omega(A\bar{P} + \frac{1}{\tau_e})} \quad \frac{\widetilde{\delta I}}{e}$$

Question 3.

Consider a photodiode with an efficiency $\eta = 0.85$ maximal at the wavelength of $1550nm$. This photodiode has a bandwidth of $20GHz$ with a load impedance of 50Ω. It works at a temperature of $300^0 K$ and an optical power of $-3dBm$ launched at its input.

Hint:

$$\frac{S}{N} = \frac{(\frac{\eta e}{h\nu})^2 P_s{}^2}{(2e(\frac{\eta e}{h\nu})P_s + \frac{4kT}{R_c})\Delta f}$$

Q 3.1 - What is the signal-to-noise ratio of this photodiode?

Firstly we know the Following	$e = 1.6 \times 10^{-19}$ C $\eta = 0.85$ $h = 6.62 \times 10^{-34}$ J s $c = 3 \times 10^8$ m/s $\lambda = 1550 \times 10^{-9}$ m $h = 6.62 \times 10^{-34}$ J s $R_c = 50$ ohms $P_s = -3dBm$ $k = 1.38x10^{-23} J/K$ Temp = 300 degrees Kelvin. $\Delta f = 20 \times 10^9$

Firstly to find v we use formula below $v = \frac{c}{\lambda}$	$\frac{3x10^{8}}{1550\text{x}10^{-9}} = 1.93548x10^{14}$
Now we can find $\frac{\eta e}{\text{hv}}$ Thus $\frac{\eta e}{\text{hv}} =$ And $(\frac{\eta e}{\text{hv}})^{2}$ =	$= \frac{(0.85)\,(1.6\text{x}10^{-19})}{(6.62\text{x}10^{-34})\,(1.935\text{x}10^{14})} = \frac{1.36\text{x}10^{-19}}{1.2809\text{x}10^{-19}}$ 1.0617 1.1272
$P_s = -3dBm$ This needs to be converted dBm to linear scale P(dB) -> (P linear scale) = P = 10 p/10	P = 10 $^{-3/10}$ => 10 $^{-0.3}$ => 0.5011 Thus Ps = 0.5
Now filling in the formula is $\frac{S}{N} =$ Filling in the formula	$\frac{(\frac{\eta e}{\text{hv}})^{2} {P_s}^{2}}{(2e\left(\frac{\eta e}{\text{hv}}\right)P_s + \frac{4kt}{R_c})\Delta f}$ $\frac{(1.1272)(0.5)^{2}}{(2)(1.6x10^{-19})(1.0617)(0.5) + \frac{4(1.38x10^{-23})300}{50})20x10^{9}}$ $= \frac{281.8\ x\ 10^{-3}}{((1.69872x10^{-19} + 3.312x10^{-22}))20x10^{9}}$

$\frac{S}{N} =$	$= \frac{281.8\ x\ 10^{-3}}{(1.702032x10^{-19})\ \ (\ 20x10^{9})}$ $= \frac{281.8\ x\ 10^{-3}}{3.404\ x\ 10^{-9}}$ $82.785\ x\ 10^{6}$
To convert linear form back to Db $\frac{S}{N} =$	P= P(dB) P $=10\log_{10}\frac{P}{1mw}$ P $=10\log_{10}\frac{82.785\ x\ 10^{6}}{1mw}$ 109.18 dBm

Q3.2 What is the value of the signal- to-noise ratio if the temperature is set at 0oC?

Using same formula replace temperature at 0'c Temp = 273	$$\frac{(\frac{\eta e}{hv})^2 {P_s}^2}{(2e\left(\frac{\eta e}{hv}\right)P_s + \frac{4kt}{R_c})\Delta f}$$ $$\frac{(1.1272)(0.5)^2}{(2)(1.6x10^{-19})(1.0617)(0.5) + \frac{4(1.38x10^{-23})273}{50})20x10^9}$$ $$= \frac{281.8\ x\ 10^{-3}}{((1.69872x10^{-19} + 3.01x10^{-22})(20x10^9)}$$ $$= \frac{281.8\ x\ 10^{-3}}{(1.701x10\ - 19)(20x10^9)}$$ $\frac{281.8\ x\ 10^{-3}}{3.403\ x10^{-9}}$ = 82.81 x 10^6
To convert linear back to Db $\frac{S}{N} =$	P= P(dB) P =10$\log_{10}\frac{P}{1mw}$ P =10$\log_{10}\frac{82.81\ x\ 10^6}{1mw}$

	104.41 dBm

Q3.3 If the temperature of a photodiode increases, what is the consequence on the signal-to-noise ratio? Justify your answer.

As the temperature increases, then the thermal noise increases. The noise level will also increase, so the overall impact is that the signal-to-noise ratio will decrease.

Q3.4 If the bandwidth of a photodiode increases, what is the consequence on the signal-to-noise ratio? Justify your answer.

Making the following assumption - any noises captured by the photodiode are independent of the frequency bandwidth of the detector (white noise).

When the bandwidth increases, this means that the photodiode can capture more noise. Hence the noise level increases and the signal-to-noise ratio will decrease.

Question 4

What is the value of the internal and external quantum efficiency of the light emitting diode. The radiative and non-radiative carrier lifetimes are 50ns and 75ns. We assume that there is no default in the semiconductor material and the contacts are not light absorbing. The refractive index of the LED is 3.6 and its emission is maximal at a wavelength of 450nm and bias current of 300 mA.

Looking for Internal and external quantum efficiency of a light emitting diode. Assume following No default in semiconductor Contacts not light absorbing	Reflective index of LED =3.6 Max emissions@ 450 nm Bias Current = 300mA Tr = 50ns Tnr = 75ns
Internal Quantum Efficiency (Must be less than 1)	Nint $= \frac{1/Tr}{\frac{1}{Tr}+\frac{1}{Tnr}}$ Now Tr = 50ns and Tnr = 75ns $= \frac{1/50ns}{\frac{1}{50ns}+\frac{1}{75ns}}$ = > $\frac{1/50ns}{33.33E06}$ => 0.6
Extraction Efficiency (Changed per email)	$\eta = \eta_{int} \bullet K \bullet \left[1-\left(\frac{n-1}{n+1}\right)^2\right] \bullet \frac{2\pi(1-\cos(\theta_c))}{4\pi}$ nint = 0.6 K = 1 n = 3.6
Need to find Θc = SIN (Θc) = $\frac{1}{n}$	n = 3.6

	SIN (Θc) = $\frac{1}{3.6}$ =16.127
	Formula n =(0.6)(1) [1 - (-$\frac{3.6-1}{3.6+1}$) 2] ($\frac{2.\pi.(1-\cos(16.126))}{4\pi}$ n= (0.6) (0.680) (0.0197) n = 0.008 N =0.8 % Another option would be to assume the internal efficiency was 1 then we could use the formula $\frac{1}{n} . \frac{1}{(n+1)2}$ which would be equal to 1.3%