

## Brian Murray, EE506 Assignment 2015

Q1: 1.1 These equations describe the evolution of the carrier number and photon number, of a laser, over time.

The symbols used are identified below:

$N$  = Carrier number

$t$  = time

$I$  = applied bias current

$e$  = |charge on electron|

$A$  = gain coefficient

$N_0$  = carrier number at transparency (when spontaneous recombination = stimulated generation)

$P$  = photon number

$\tau_c$  = carrier lifetime

$\tau_p$  = photon lifetime

$\beta$  = spontaneous emission coupling fraction

1.2 Rate equations:

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_c} \quad (1)$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + \beta \frac{N}{\tau_c} \quad (2)$$

• Steady-state  $\Rightarrow \frac{dN}{dt} = \frac{dP}{dt} = 0$

• Assumption: Stimulated emission  $\gg$  spontaneous emission at threshold  
 $\Rightarrow A(N - N_0)P \gg \beta \frac{N}{\tau_c}$ , so let  $\beta \frac{N}{\tau_c} \rightarrow 0$

Then, using (2) we get:  $A(N - N_0)P = \frac{P}{\tau_p} \Rightarrow N = N_0 + \frac{1}{A\tau_p}$

Fill this into (1):

$$\frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_c} = 0$$

$$\Rightarrow \frac{I}{e} - A\left(N_0 + \frac{1}{A\tau_p} - N_0\right)P - \frac{N_0 + \frac{1}{A\tau_p}}{\tau_c} = 0$$

$$\Rightarrow I = \frac{e}{\tau_p} P + \frac{e}{\tau_c} \left(N_0 + \frac{1}{A\tau_p}\right) \text{ at threshold.}$$

But,  $P > 0$  only after threshold is reached, and so set  $P = 0$ :


$$\therefore I_{th} = \frac{e}{\tau_c} \left(N_0 + \frac{1}{A\tau_p}\right)$$

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Use the other document.

1.3: Above threshold, the carrier number is clamped to  $N = N_0 + \frac{I}{A \Gamma_p}$  as the gain equals the losses. Any excess carriers are used directly for stimulated emission.

Below threshold, the carrier number increases linearly with current, according to  $N = \left(\frac{\tau_0}{e}\right) I$ .



Q2: 2.i Rate eqns:  $\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e}$   
 $\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p} + \beta \frac{N}{\tau_e}$

Neglect spontaneous emission, so  $\beta \frac{N}{\tau_e} \rightarrow 0$ . 1

Apply a small perturbation:

$$\frac{d(\bar{N} + \delta N)}{dt} = \frac{I + \delta I}{e} - A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{N} + \delta N}{\tau_e}$$

$$\frac{d(\bar{P} + \delta P)}{dt} = A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \left(\frac{\bar{P} + \delta P}{\tau_p}\right)$$

After multiplying this out, we set all second order terms to zero  
 i.e.  $\delta N \delta P \rightarrow 0$  2

Also we note that  $\frac{d\bar{N}}{dt}$  and  $\frac{d\bar{P}}{dt} = 0$ , still 3

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(\bar{N} - N_0)\delta P - \left(\bar{A}P + \frac{1}{\tau_e}\right)\delta N$$

$$\frac{d(\delta P)}{dt} = \left[A(\bar{N} - N_0) - \frac{1}{\tau_p}\right]\delta P + A\bar{P}\delta N$$

Now recall  $A(\bar{N} - N_0) = \frac{1}{\tau_p}$  as we are above threshold and so  
 the gain = losses.


$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \delta P \quad \text{4}$$

$$\frac{d(\delta P)}{dt} = A\bar{P}\delta N$$

Then writing this in matrix form:


$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \underbrace{\begin{pmatrix} -\left(\bar{A}P + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix}}_{= M} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$
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good

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right!

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good.

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missing tp

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2.2:  $M$  describes the rate of change of  $\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$  a perturbation in the carrier and photon numbers of the laser due to a perturbation in current.

$A$  = Gain constant

$\bar{P}$  = photon number

$\tau_c$  = ~~carrier~~ carrier lifetime  $\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$  2

$\tau_p$  = photon lifetime

~~2.3: In the frequency domain (after FT), the matrix elements in 2.2 are invariant~~

2.3: The elements of  $M$  are invariant under Fourier transformation.  
Apply FT:

$$\begin{pmatrix} i\omega \tilde{S}_N \\ i\omega \tilde{S}_P \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{\tau_c}) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \tilde{S}_N \\ \tilde{S}_P \end{pmatrix} + \begin{pmatrix} \tilde{S}_I/e \\ 0 \end{pmatrix}$$

A little factorisation gives:

$$\begin{pmatrix} i\omega + (A\bar{P} + \frac{1}{\tau_c}) & \frac{1}{\tau_p} \\ -A\bar{P} & i\omega \end{pmatrix} \begin{pmatrix} \tilde{S}_N \\ \tilde{S}_P \end{pmatrix} = \begin{pmatrix} \tilde{S}_I/e \\ 0 \end{pmatrix}$$

→ Inverse of this:  $\frac{1}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_c})} \begin{pmatrix} -i\omega & \frac{1}{\tau_p} \\ -A\bar{P} & -i\omega - (A\bar{P} + \frac{1}{\tau_c}) \end{pmatrix}$   $\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$  3


Then we ~~see~~ see our answer:

$$\tilde{S}_P = \frac{-A\bar{P}}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_c})} \frac{\tilde{S}_I}{e}$$


$$\tilde{S}_N = \frac{-i\omega}{\omega^2 - \frac{A\bar{P}}{\tau_p} - i\omega(A\bar{P} + \frac{1}{\tau_c})} \frac{\tilde{S}_I}{e}$$
 $\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$  4

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M is a 2x2 matrix.


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just express M as seen above  
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good

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Q3: 3-1: 
$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_c}\right)\Delta f}$$

$\eta = 0.85$

$\lambda_{\text{max power}} = 1550 \text{ nm} \Rightarrow \nu = \frac{c}{\lambda} = \cancel{1.935 \times 10^{14}} \text{ Hz}$

$\Delta f = 20 \text{ GHz}$

$R_c = 50 \Omega$

$T = 300 \text{ K}$

$P = -3 \text{ dBm} = \frac{10^{\left(\frac{-3}{10}\right)} \text{ Watts}}{1000} = 0.0005 \text{ Watts}$

Input into Wolfram Mathematica!

Result:  $\frac{S}{N} = 28218.9$

$\Rightarrow \frac{S}{N} = 44.5 \text{ dB}$  1

3-2: Now  $T = 0^\circ\text{C} = 273 \text{ K}$


Put it back into Mathematica:

Result:  $\frac{S}{N} = 30002.5$


$\Rightarrow \frac{S}{N} = 44.8 \text{ dB}$  2

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## Brian Murray, EE506 Assignment 2015, Question 3

Setting the parameters:

$$h = 6.62 \times 10^{-34};$$

$$v = 0.85;$$

$$P_{\text{dBm}} = -3;$$

$$T_1 = 300;$$

$$R_c = 50;$$

$$\Delta f = 20 \times 10^9;$$

$$c = 3 \times 10^8;$$

$$e = 1.6 \times 10^{-19};$$

$$k = 1.38 \times 10^{-23};$$

$$\lambda = 1550 \times 10^{-9};$$

Formula for converting power in watts to power in dBm:

$$P_W = 10^{(P_{\text{dBm}}/10)} / 1000;$$

### ■ Question 3.1

Calculating the signal-to-noise ratio in a linear scale:

$$N \left[ \frac{\left( \frac{v e}{h c / \lambda} \right)^2 (P_W)^2}{\left( 2 e \left( \frac{v e}{h c / \lambda} \right) (P_W) + \frac{4 k T_1}{R_c} \right) (\Delta f)} \right]$$

$$28218.9$$

Calculating the signal-to-noise ratio in dB:

$$P_{\text{dB}} = 10 \text{ Log}_{10} [\%]$$

$$44.5054$$

### ■ Question 3.2:

Setting the new temperature in kelvin:

$$T_2 = 273.15;$$

Calculating the signal-to-noise ratio in a linear scale:

$$N \left[ \frac{\left( \frac{v e}{h c / \lambda} \right)^2 (P_W)^2}{\left( 2 e \left( \frac{v e}{h c / \lambda} \right) (P_W) + \frac{4 k T_2}{R_c} \right) (\Delta f)} \right]$$

$$29991.9$$

Calculating the signal-to-noise ratio in dB:

$$P_{dB} = 10 \text{ Log}_{10} [\%]$$

-44.77



Q4:

4.1:

Internal quantum efficiency:  $\eta_{int} = \frac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}}$

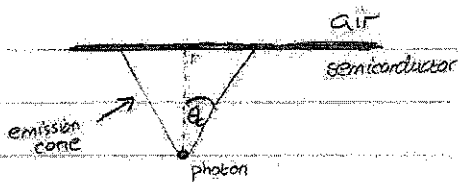
$$\Rightarrow \eta_{int} = \frac{50e9}{50e9 + 75e9}$$

$$= \frac{50}{50+75}$$

$$\Rightarrow \underline{\eta_{int}} = \underline{\frac{2}{5}} = \underline{.4}$$

~~Internal quantum efficiency~~

Extraction efficiency: There are two factors we need to take into account:



$\theta_c =$  critical angle

First, we find the number of photons that exit through this ~~emission~~ <sup>emission</sup> cone (i.e. those that are not totally internally reflected.)

This will be the ratio of the solid angle subtended by the photon to the solid angle through which all the photons can travel (i.e., a full sphere.)

$$\therefore \eta_{ext1} = \frac{2\pi \int_0^{\theta_c} \sin\theta d\theta}{4\pi} = \frac{1}{2}(1 - \cos\theta_c)$$

Now use  $\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{sc}}\right) = \sin^{-1}\left(\frac{1}{3.6}\right) = .281 \text{ rad} = 16.13^\circ$

$$\therefore \eta_{ext1} = \frac{1}{2}(1 - \cos(16.13^\circ)) = .0197$$

The second factor involves the partial reflection of the photons' energy at the sc-air interface. (Treat photon as a wave and use transmission and reflection coefficients).

PTO

wrong calculation.  
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The reflectance of the facets is given by #

$$T = \frac{n_{air} \cos \theta_i}{n_{sc} \cos \theta_i} t^2 \quad \text{where } \theta_i = \text{photon incident angle}$$

$$\text{and } t = \frac{2 n_{sc} \cos \theta_i}{n_{air} \cos \theta_i + n_{sc} \cos \theta_i}$$

$$\text{Thus, } t = \frac{2(3.6)}{1 + 3.6} = 1.565$$

and then,

$$T = \frac{1}{3.6} (1.565)^2 = .68$$

So, finally, the extraction efficiency is

$$\underline{\eta_{\text{extraction}}} = \eta_{\text{ext}2} \cdot T = (.0197)(.68) = \underline{\underline{.013}}$$





