SCHOOL OF ELECTRONICS ENGINEERING

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EE506: Fundamentals of Photonics Assignment

Programme/Major:	M.eng Electronic Engineering/Semiconductors
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Question 1

1.1

$N = Carrier\ Number$
$I = biased\ laser\ current,$
$A = gain\ coefficient$
$N_0 = \ carrier\ number\ at\ transparency$
$P = Photon\ number$
$T_e = Carrier\ lifetime$
$T_p = Photon\ lifetime$
$\beta = spontaneous\ emission\ coupling$

1.2

Conditions for threshold current
dN/dt = 0 and dP/dt = 0, GP>> $\beta N/T_e$

The spontaneous emission component ($\beta \frac{N}{T_e}$) is neglected

So: $$N = N_0 + \frac{1}{AT_p}$$

$$P = \frac{T_p}{e}(I - I_s)$$

I_s is the threshold current:

$$I_s = \frac{e}{T_e}(N_0 + \frac{1}{AT_p})$$

$$I_s = \frac{1.6 \times 10^{-19}}{2.2 \times 10^{-9}}(1.2 \times 10^8 + \frac{1}{8 \times 10^3 \times 1.6 \times 10^{-12}})$$

$$I_s = 0.0144\ A$$

1.3

Above threshold: Carrier Number (N) corresponds to the number of carriers when the gain is equal to the losses, P increases with the current but there is no change in N

$$N = N_0 + \frac{1}{AT_p}$$

Below threshold: Gain is zero, so: $A(N - N_0)P = 0$, when this is applied to the carrier number equation

$$N = \frac{T_e}{e}I$$

Question 2

2.1 The spontaneous emission component ($\beta \frac{N}{T_e}$) is neglected

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{T_p}$$
$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{T_e}$$

Applying perturbation condition: $I = \bar{I} + \delta I, \quad P = \bar{P} + \delta P, \quad N = \bar{N} + \delta N$

So:

$$\frac{d(\bar{P} + \delta P)}{dt} = A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{P} + \delta P}{T_p}$$

$$\frac{d(\bar{N} + \delta N)}{dt} = \frac{\bar{I} + \delta I}{e} - A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{N} + \delta N}{T_e}$$

So:

$$\frac{d\bar{P}}{dt} + \frac{d\delta P}{dt} = A(\bar{N} - N_0)\bar{P} - \frac{\bar{P}}{T_p} + A(\bar{N} - N_0)\delta P + A\bar{P}\delta N + A\delta N\delta P - \frac{\delta P}{T_p}$$

$$\frac{d\bar{N}}{dt} + \frac{d\delta N}{dt} = \frac{\bar{I}}{e} - A(\bar{N} - N_0)\bar{P} - \frac{\bar{N}}{T_e} + \frac{\delta I}{e} - A(\bar{N} - N_0)\delta P - A\bar{P}\delta N - A\delta N\delta P - \frac{\delta N}{T_e}$$

All 2nd order terms are ignored, and $A(\bar{N} - N_0) = \frac{1}{T_p}$

So:

$$\frac{d\delta P}{dt} = \frac{\bar{P}}{T_p} - \frac{\bar{P}}{T_p} + \frac{\delta P}{T_p} + A\bar{P}\delta N - \frac{\delta P}{T_p}$$

$$\frac{d\delta N}{dt} = \frac{\bar{I}}{e} - \frac{\bar{P}}{T_p} - \frac{\bar{N}}{T_e} + \frac{\delta I}{e} - \frac{\delta P}{T_p} - A\bar{P}\delta N - \frac{\delta N}{T_e}$$

So:

$$\frac{d\delta P}{dt} = A\bar{P}\delta N$$
$$\frac{d\delta N}{dt} = \frac{\delta I}{e} - \frac{\delta P}{T_p} - (A\bar{P} + \frac{1}{T_e})\delta N$$

So:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta i/e \\ 0 \end{pmatrix}$$

2.2 Elements of the matrix $M = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix}$

$A = Gain\ coefficient$ $\bar{P} =\ Average\ number\ of\ photon$

$T_p = Photon\ lifetime$ $T_e = Carrier\ lifetime$

2.3 The signal can be rewritten as

$$\frac{d}{dt}\begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix}\begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta i/e \\ 0 \end{pmatrix}$$

Converting to Fourier domain

$$\begin{pmatrix} j\Omega\widetilde{\delta N} \\ j\Omega\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix}\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta i/e} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} j\Omega + (A\bar{P} + \frac{1}{T_e}) & \frac{1}{T_p} \\ -A\bar{P} & j\Omega \end{pmatrix}\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta \imath/e} \\ 0 \end{pmatrix}$$

Assuming, M = $\begin{pmatrix} j\Omega + (A\bar{P} + \frac{1}{T_e}) & \frac{1}{T_p} \\ -A\bar{P} & j\Omega \end{pmatrix}$, V = $\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix}$ and B = $\begin{pmatrix} \widetilde{\delta \imath/e} \\ 0 \end{pmatrix}$

$V = M^{-1}B$

$$M^{-1} = \frac{1}{\left(\left(j\Omega + \left(A\bar{P} + \frac{1}{T_e}\right)\right) * J\Omega\right) - \left(\frac{1}{T_p} * -A\bar{P}\right)}\begin{pmatrix} j\Omega & -\frac{1}{T_p} \\ A\bar{P} & j\Omega + (A\bar{P} + \frac{1}{T_e}) \end{pmatrix}$$

N:B: $\frac{A\bar{P}}{T_p} = \omega_r^2$, $\left(A\bar{P} + \frac{1}{T_e}\right) = \frac{2}{T_r}$,

$$M^{-1} = \frac{1}{-\Omega^2 + J\Omega\frac{2}{T_r} - \omega_r^2}\begin{pmatrix} j\Omega & -\frac{1}{T_p} \\ \omega_r^2 T_p & j\Omega + \frac{2}{T_r} \end{pmatrix}$$

Since: $\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1}\begin{pmatrix} \widetilde{\delta \imath/e} \\ 0 \end{pmatrix}$

Then:

$$\widetilde{\delta N} = \frac{j\Omega}{-\Omega^2 + J\Omega\frac{2}{T_r} - \omega_r^2}\frac{\widetilde{\delta \imath}}{e} \quad and \quad \widetilde{\delta P} = \frac{\omega_r^2 T_p}{-\Omega^2 + J\Omega\frac{2}{T_r} - \omega_r^2}\frac{\widetilde{\delta \imath}}{e}$$

Question 3

3.1

$$v = \frac{c}{\gamma} = \frac{3 \times 10^8}{1550 \times 10^{-9}} = 1.94 \times 10^{14}, \; P_s = 501 \times 10^{-6}\, W$$

$$\frac{S}{N}: \frac{\left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 1.94 \times 10^{14}}\right)^2 * (501 \times 10^{-6})^2}{\left(\left((2 \times 1.6 \times 10^{-19}) * \left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 1.94 \times 10^{14}}\right) * 501 \times 10^{-6}\right) + \frac{4 \times 1.38 \times 10^{-23} \times 300}{50}\right) * 20 \times 10^9}$$

$$\frac{S}{N} = 44.5\, dBm$$

3.2 If temperature is at 0°c = 273k

$$\frac{S}{N}: \frac{\left(\frac{0.85 \times 1.6 \times 10^{-19}}{1.3 \times 10^{-19}}\right)^2 * (1.737 \times 10^{-4})^2}{\left(\left((2 \times 1.6 \times 10^{-19}) * \left(\frac{0.85 \times 1.6 \times 10^{-19}}{1.3 \times 10^{-19}}\right) * 1.737 \times 10^{-4}\right) + \frac{4 \times 1.38 \times 10^{-23} \times 273}{50}\right) * 20 \times 10^9}$$

$$\frac{S}{N} = 45\, dBm$$

3.3
Temperature increase would result in decrease in the value of the signal-noise-ratio.
The temperature increases produces more noise which results in a lower signal-noise-ratio.

3.4
Bandwidth increase would result in decrease in the value of the signal-noise-ratio.
As the noise spreads out over all frequencies, the wider the bandwidth of the device, the greater the noise level. And an increase in noise level results in a lower signal-noise-ratio.

Question 4

Internal Quantum Efficiency: n_{int}

$$\text{Carrier lifetime: } \tau = \frac{\tau_r \tau_{nr}}{\tau_r + \tau_{nr}} = \frac{50 \times 75}{50+75} = 30ns$$

$$n_{int} = \frac{\tau}{\tau_r} = \frac{30}{50} = 0.6$$

Extraction efficiency: n_e

$$n_e = \left[\frac{1}{(n+1)^2}\right] \times \frac{1}{n}$$

$$n_e = \left[\frac{1}{(3.6+1)^2}\right] \times \frac{1}{3.6}$$

$$n_e = 0.013$$

External Quantum Efficiency: n_{ext}

$$n_{ext} = n_e \times n_{int}$$

$$n_{ext} = 0.013 \times 0.6$$

$$n_{ext} = 0.0079$$