



SCHOOL OF ELECTRONICS ENGINEERING

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EE506: Fundamentals of Photonics Assignment

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Question 1

1.1

N = Carrier Number
 I = biased laser current,
 A = gain coefficient
 N_0 = carrier number at transparency
 P = Photon number
 T_e = Carrier lifetime
 T_p = Photon lifetime
 β = spontaneous emission coupling



1.2

Conditions for threshold current
 $dN/dt = 0$ and $dP/dt = 0$, $GP \gg \beta N/T_e$

The spontaneous emission component ($\beta \frac{N}{T_e}$) is neglected

So:

$$N = N_0 + \frac{1}{AT_p}$$
$$P = \frac{T_p}{e}(I - I_s)$$

I_s is the threshold current:

$$I_s = \frac{e}{T_e} \left(N_0 + \frac{1}{AT_p} \right)$$

$$I_s = \frac{1.6 \times 10^{-19}}{2.2 \times 10^{-9}} \left(1.2 \times 10^8 + \frac{1}{8 \times 10^3 \times 1.6 \times 10^{-12}} \right)$$

$$I_s = 0.0144 \text{ A}$$



1.3

Above threshold: Carrier Number (N) corresponds to the number of carriers when the gain is equal to the losses, P increases with the current but there is no change in N

$$N = N_0 + \frac{1}{AT_p}$$



Below threshold: Gain is zero, so: $A(N - N_0)P = 0$, when this is applied to the carrier number equation

$$N = \frac{T_e}{e} I$$



Question 2

2.1 The spontaneous emission component ($\beta \frac{N}{T_e}$) is neglected

$$\begin{aligned}\frac{dP}{dt} &= A(N - N_0)P - \frac{P}{T_p} \\ \frac{dN}{dt} &= \frac{I}{e} - A(N - N_0)P - \frac{N}{T_e}\end{aligned}$$

Applying perturbation condition: $I = \bar{I} + \delta I$, $P = \bar{P} + \delta P$, $N = \bar{N} + \delta N$
So:

$$\frac{d(\bar{P} + \delta P)}{dt} = A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{P} + \delta P}{T_p}$$

$$\frac{d(\bar{N} + \delta N)}{dt} = \frac{\bar{I} + \delta I}{e} - A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{\bar{N} + \delta N}{T_e}$$

So:

$$\frac{d\bar{P}}{dt} + \frac{d\delta P}{dt} = A(\bar{N} - N_0)\bar{P} - \frac{\bar{P}}{T_p} + A(\bar{N} - N_0)\delta P + A\bar{P}\delta N + A\delta N\delta P - \frac{\delta P}{T_p}$$

$$\frac{d\bar{N}}{dt} + \frac{d\delta N}{dt} = \frac{\bar{I}}{e} - A(\bar{N} - N_0)\bar{P} - \frac{\bar{N}}{T_e} + \frac{\delta I}{e} - A(\bar{N} - N_0)\delta P - A\bar{P}\delta N - A\delta N\delta P - \frac{\delta N}{T_e}$$

All 2nd order terms are ignored, and $A(\bar{N} - N_0) = \frac{1}{T_p}$

So:

$$\frac{d\delta P}{dt} = \frac{\bar{P}}{T_p} - \frac{\bar{P}}{T_p} + \frac{\delta P}{T_p} + A\bar{P}\delta N - \frac{\delta P}{T_p}$$

$$\frac{d\delta N}{dt} = \frac{\bar{I}}{e} - \frac{\bar{P}}{T_p} - \frac{\bar{N}}{T_e} + \frac{\delta I}{e} - \frac{\delta P}{T_p} - A\bar{P}\delta N - \frac{\delta N}{T_e}$$




So:

$$\begin{aligned}\frac{d\delta P}{dt} &= A\bar{P}\delta N \\ \frac{d\delta N}{dt} &= \frac{\delta I}{e} - \frac{\delta P}{T_p} - (A\bar{P} + \frac{1}{T_e})\delta N\end{aligned}$$

So:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$



2.2 Elements of the matrix $M = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix}$ 

$A =$ Gain coefficient

$\bar{P} =$ Average number of photon

$T_p =$ Photon lifetime

$T_e =$ Carrier lifetime

2.3 The signal can be rewritten as

$$\frac{d}{dt} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta i/e \\ 0 \end{pmatrix}$$

Converting to Fourier domain


$$\begin{pmatrix} j\Omega \widetilde{\delta N} \\ j\Omega \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(A\bar{P} + \frac{1}{T_e}) & -\frac{1}{T_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta i/e} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} j\Omega + (A\bar{P} + \frac{1}{T_e}) & \frac{1}{T_p} \\ -A\bar{P} & j\Omega \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta i/e} \\ 0 \end{pmatrix}$$

Assuming, $M = \begin{pmatrix} j\Omega + (A\bar{P} + \frac{1}{T_e}) & \frac{1}{T_p} \\ -A\bar{P} & j\Omega \end{pmatrix}$, $V = \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix}$ and $B = \begin{pmatrix} \widetilde{\delta i/e} \\ 0 \end{pmatrix}$

$V = M^{-1}B$


$$M^{-1} = \frac{1}{\left((j\Omega + (A\bar{P} + \frac{1}{T_e})) * j\Omega \right) - \left(\frac{1}{T_p} * -A\bar{P} \right)} \begin{pmatrix} j\Omega & -\frac{1}{T_p} \\ A\bar{P} & j\Omega + (A\bar{P} + \frac{1}{T_e}) \end{pmatrix}$$

N:B: $\frac{A\bar{P}}{T_p} = \omega_r^2$, $(A\bar{P} + \frac{1}{T_e}) = \frac{2}{T_r}$ 

$$M^{-1} = \frac{1}{-\Omega^2 + j\Omega \frac{2}{T_r} - \omega_r^2} \begin{pmatrix} j\Omega & -\frac{1}{T_p} \\ \omega_r^2 T_p & j\Omega + \frac{2}{T_r} \end{pmatrix}$$

Since: $\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1} \begin{pmatrix} \widetilde{\delta i/e} \\ 0 \end{pmatrix}$

Then:

$$\widetilde{\delta N} = \frac{j\Omega}{-\Omega^2 + j\Omega \frac{2}{T_r} - \omega_r^2} \frac{\widetilde{\delta i}}{e} \quad \text{and} \quad \widetilde{\delta P} = \frac{\omega_r^2 T_p}{-\Omega^2 + j\Omega \frac{2}{T_r} - \omega_r^2} \frac{\widetilde{\delta i}}{e}$$
 



Question 3

3.1

$$v = \frac{c}{\gamma} = \frac{3 \times 10^8}{1550 \times 10^{-9}} = 1.94 \times 10^{14}, P_s = 501 \times 10^{-6} W$$

$$\frac{S}{N} = \frac{\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 1.94 \times 10^{14}})^2 * (501 \times 10^{-6})^2}{\left((2 \times 1.6 \times 10^{-19}) * \left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 1.94 \times 10^{14}} \right) * 501 \times 10^{-6} \right) + \frac{4 \times 1.38 \times 10^{-23} \times 300}{50}} * 20 \times 10^9} = 44.5 \text{ dBm}$$

3.2 If temperature is at 0°C = 273K

$$\frac{S}{N} = \frac{\frac{0.85 \times 1.6 \times 10^{-19}}{1.3 \times 10^{-19}})^2 * (1.737 \times 10^{-4})^2}{\left((2 \times 1.6 \times 10^{-19}) * \left(\frac{0.85 \times 1.6 \times 10^{-19}}{1.3 \times 10^{-19}} \right) * 1.737 \times 10^{-4} \right) + \frac{4 \times 1.38 \times 10^{-23} \times 273}{50}} * 20 \times 10^9} = 45 \text{ dBm}$$

3.3

Temperature increase would result in decrease in the value of the signal-noise-ratio.

The temperature increase produces more noise which results in a lower signal-noise-ratio.

3.4

Bandwidth increase would result in decrease in the value of the signal-noise-ratio.

As the noise spreads out over all frequencies, the wider the bandwidth of the device, the greater the noise level. And an increase in noise level results in a lower signal-noise-ratio.

Question 4

Internal Quantum Efficiency: n_{int}

$$\text{Carrier lifetime: } \tau = \frac{\tau_r \tau_{nr}}{\tau_r + \tau_{nr}} = \frac{50 \times 75}{50 + 75} = 30 \text{ ns}$$

$$n_{int} = \frac{\tau}{\tau_r} = \frac{30}{50} = 0.6$$

Extraction efficiency: n_e

$$n_e = \left[\frac{1}{(n + 1)^2} \right] \times \frac{1}{n}$$

$$n_e = \left[\frac{1}{(3.6 + 1)^2} \right] \times \frac{1}{3.6}$$

$$n_e = 0.013$$

External Quantum Efficiency: n_{ext}

$$n_{ext} = n_e \times n_{int}$$

$$n_{ext} = 0.013 \times 0.6$$

$$n_{ext} = 0.0079$$

