Appendix A Student Declaration of Academic Integrity

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Declaration

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Programme: M.Eng Electronic Systems

Module Code: EE506

Assignment Title: EE506 Assignment 2015

Submission Date: 24/04/2015

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

I have identified and included the source of all facts, ideas, opinions, viewpoints of others in the assignment references. Direct quotations from books, journal articles, internet sources, module text, or any other source whatsoever are acknowledged and the sources cited are identified in the assignment references.

I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

I have read and understood the DCU library referencing guidelines (available at: http://www.library.dcu.ie/LibraryGuides/DCU Library Guide to Harvard Style of Citing & Referencing/player.html Referencing/player.html accessed 9th January 2013) and/or recommended in the assignment guidelines and/or programme documentation.

By signing this form or by submitting this material online I confirm that this assignment, or any part of it, has not been previously submitted by me or any other person for assessment on this or any other course of study.

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Date: 24/04/2015

Report of Suspected Plagiarism / Breach of Academic Integrity

EE506: Assignment 2015

Name: Saumya George Palissery

Question 1:

$$\begin{cases} \frac{dN}{dt} = \frac{I}{e} - A(N - No)P - \frac{N}{\tau_e} \\ \frac{dP}{dt} = A(N - No)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e} \end{cases}$$

1.1.

The rate equations are the two sets of differential equations that represent the time evolution of the **laser** that is **single mode**. Both equations are to be solved simultaneously, where one represents the **carrier number** and the other represents the **photon number**.

The first equation describes the **Carrier time evolution** $\frac{dN}{dt}$. The term $\frac{I}{e}$ describes **the carriers that are injected by the bias current per unit of time**. The ratio represents the number of charges per unit time divided by the elementary unit charge. **A** is the **slope of the linear function of the carrier, N** is **the carrier number** and N_o is **the carrier number** that is required, to have no attenuation and no amplification of the signal. **P** is the photon number and $\frac{N}{\tau_e}$ **is the carriers that recombine or get lost in the crystal** and τ_e **is the lifetime of the carrier** before it recombines**.**

$\frac{dP}{dt}$ is **the photon time evolution and where** $\frac{P}{\tau_P}$ is **the rate of photons lost** by the transmission of the facets and by imperfections of the waveguide. **β** is **the spontaneous emission coupled to the mode.**

1.2.

The expression for the current threshold is obtained by considering the case where $\gg \frac{\beta N}{\tau_e}$, and spontaneous emission is neglected as more photons are generated by stimulated emission. It is assumed that the carrier number N_o is at transparency. The linear gain $\boldsymbol{G = A(N - N_o)}$**.**

In steady state as there is no time variation, $\frac{dP}{dt} = 0$. We use the second differential equation for $\frac{dP}{dt}$ to determine the value of the threshold current.

$$0 = GP + \beta\frac{N}{\tau_e} - \frac{P}{\tau_P}$$

$$0 = A(N - N_o)P + \beta\frac{N}{\tau_e} - \frac{P}{\tau_P}$$

Dividing both sides by P:

$$0 = A(N - N_o) + \beta\frac{N}{\tau_e P} - \frac{1}{\tau_P}$$

$$0 = A(N - N_o) + \cancel{\beta\frac{N}{\tau_e P}} - \frac{1}{\tau_P}$$

The $\beta\frac{N}{\tau_e P}$ is cancelled out as it represents the spontaneous emission which is negligible.

$$\frac{1}{\tau_p} = A(N - N_o)$$

$$N = N_o + \frac{1}{A\tau_p}$$

Now take the first differential equation where $\frac{dN}{dt}$ = 0.

$$0 = \frac{I}{e} - A(N - N_o)P - \frac{N}{\tau_e}$$

$$\frac{I}{e} - \frac{N}{\tau_e} = A(N - N_o)P$$

Subbing the value of N into the first differential equation of the carrier number gives the following result:

$$\frac{I}{e} - \frac{1}{\tau_e}\left(N_o + \frac{1}{A\tau_p}\right) = \frac{P}{\tau_p}$$

$$P = \frac{\tau_p}{e}\left(I - \left(\frac{N_o}{\tau_e} + \frac{1}{\tau_e A\,\tau_p}\right)\right)$$

Now, P can be written as where I_s is the threshold current: $P = \frac{\tau_p}{e}(I - I_s)$

So, the **threshold current I_s** is given as follows:

$$I_s = \frac{e}{\tau_e}\left(N_o + \frac{1}{A\tau_p}\right)$$

$e = 1.6 * 10^{-19}\ C$

$N_o = 1.2 * 10^8$

$A = 8 * 10^3\ s^{-1}$

$\tau_p = 1.6\ ps$

$\tau_e = 2.2\ ns$

$$I_s = \frac{1.6 * 10^{-19}}{2.2 * 10^{-9}}\left(1.2 * 10^8 + \frac{1}{(8 * 10^3)(1.6 * 10^{-12})}\right) = 14.409\ mA$$

1.3.

Above the threshold the carrier number is clamped at $N_o + \frac{1}{A\tau_p}$ because the value of N is fixed and is independent of the bias current. Below the threshold, the value of the carrier number N increases along with the increasing bias current in a linear manner where $\frac{dN}{dt} = 0$ and the photon number P is negligible and $= \frac{\tau_e I}{e}$.

Question 2:

2.1.

The rate equations are as follows:

$$\frac{dP}{dt} = A\,(N - N_o)P - \frac{P}{\tau_p}$$

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_o)P - \frac{N}{\tau_e}$$

When the bias current is above the threshold current:

$$I = \underline{I} + \delta I$$

$$N = \underline{N} + \delta N$$

$$P = \underline{P} + \delta P$$

The change in current **I** causes a change in photon number **P** and carrier number **N**. The spontaneous emission term $\frac{\beta N}{\tau_e}$ is neglected as it is in steady state and above threshold, $(\underline{N} - N_o) = \frac{1}{\tau_p}$.

Substituting them into the rate equations gives us the following:

$$\frac{d(\underline{N} + \delta N)}{dt} = \frac{(\underline{I} + \delta I)}{e} - A\big((\underline{N} + \delta N) - N_o\big)(\underline{P} + \delta P) - \frac{(\underline{N} + \delta N)}{\tau_e}$$

$$\frac{d(\underline{P} + \delta P)}{dt} = A\,\Big((\underline{N} + \delta N) - N_o\Big)(\underline{P} + \delta P) - \frac{(\underline{P} + \delta P)}{\tau_p}$$

The equations for perturbation are as follows where second order terms were neglected:

$$\frac{d(\delta N)}{dt} = \frac{(\delta I)}{e} - A(\underline{N} - N_o)(\delta P) - (A\underline{P} + \frac{1}{\tau_e})(\delta N)$$

$$\frac{d(\delta P)}{dt} = A\left(\underline{N} - N_o\right) - \frac{1}{\tau_p}(\delta P) + A\underline{P}\delta N$$

These equations can also be written as:

$$\frac{d(\delta N)}{dt} = \frac{(\delta I)}{e} - \frac{(\delta P)}{\tau_p} - (A\underline{P} + \frac{1}{\tau_e})(\delta N)$$

$$\frac{d(\delta P)}{dt} = A\,\underline{P}\delta N$$

In the form of matrices, the equations are:

$$\begin{pmatrix} \frac{d(\delta N)}{dt} \\ \frac{d(\delta P)}{dt} \end{pmatrix} = \begin{pmatrix} -\left(A\underline{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\underline{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{dI}{e} \\ 0 \end{pmatrix}$$

Therefore, the matrix $M = \begin{pmatrix} -\left(A\underline{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\underline{P} & 0 \end{pmatrix}$

2.2.

$M = \begin{pmatrix} -\left(A\underline{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\underline{P} & 0 \end{pmatrix}$

2.3.

The elements of the matrix are invariant in the Fourier domain.

$$\begin{pmatrix} j\omega\widetilde{\delta N} \\ j\omega\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -\left(A\underline{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\underline{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \frac{\widetilde{dI}}{e} \\ 0 \end{pmatrix}$$

$$M\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \frac{\widetilde{dI}}{e} \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} j\omega + \left(A\underline{P} + \frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\underline{P} & j\omega \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1}\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix}$$

$$M^{-1} = \frac{1}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega(A\underline{P} + \frac{1}{\tau_e})} \begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\underline{P} & -j\omega - (A\underline{P} + \frac{1}{\tau_e}) \end{pmatrix}$$

$$\omega_R^2 = \frac{A\underline{P}}{\tau_p}$$

And also,

$\frac{2}{\tau_R} = \left(A\underline{P} + \frac{1}{\tau_e}\right)$. ω_R is the relaxation oscillation and $\frac{2}{\tau_R}$ is the dumping coefficient.

$$\widetilde{\delta P} = \frac{-A\underline{P}}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)} \left(\frac{\widetilde{dI}}{e}\right)$$

$$\frac{\widetilde{\delta P}}{\widetilde{\delta I}} = \frac{\frac{-\tau_p \omega_R^2}{e}}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)}$$

$$\widetilde{\delta N} = \frac{-j\omega}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)} \left(\frac{\widetilde{dI}}{e}\right)$$

Question 3:

3.1.

$$\frac{S}{N} = \frac{\left(\frac{0.85\, e}{\frac{hc}{\lambda}}\right)^2 P_S^2}{\left(2e\left(\frac{0.85\, e}{\frac{hc}{\lambda}}\right) P_S + \frac{4kT}{R_C}\right)\Delta f} = dB$$

$$\frac{S}{N} = \frac{\left(\frac{0.85\,(1.6 * 10^{-19})}{\frac{(6.62 * 10^{-34})(3 * 10^8)}{1550 * 10^9}}\right)^2 (0.000501187)^2}{\left(2(1.6 * 10^{-19})\left(\frac{0.85\,(1.6 * 10^{-19})}{\frac{(6.62 * 10^{-34})(3 * 10^8)}{1550 * 10^9}}\right)(0.000501187) + \frac{4(1.38 * 10^{-23})(300)}{50}\right)(20 * 10^9)}$$

$$\frac{S}{N} = 44.505\ dB$$

3.2.

Temperature = 0^0 C = 273 K

$$\frac{S}{N}$$

$$= \frac{\left(\frac{0.85\ (1.6 * 10^{-19})}{\frac{(6.62 * 10^{-34})(3 * 10^8)}{1550 * 10^9}}\right)^2 (0.000501187)^2}{\left(2(1.6 * 10^{-19})\left(\frac{0.85\ (1.6 * 10^{-19})}{\frac{(6.62 * 10^{-34})(3 * 10^8)}{1550 * 10^9}}\right)(0.000501187) + \frac{4(1.38 * 10^{-23})273}{50}\right)(20 * 10^9)}$$

$$\frac{S}{N} = 44.77\ dB$$

3.3.

The SNR for the photodiode increases with decreasing temperature.

3.4.

The SNR for the photodiode decreases along with increasing temperature.

Question 4:

4.1.

Internal Quantum Efficiency of an LED = $\frac{\tau_{nr}}{\tau_r+\tau_{nr}}$

Where τ_r the radiative recombination and τ_{nr} is the non- radiative recombination time.

$$\eta_{internal} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}} = \frac{75 * 10^{-9}}{(50 * 10^{-9}) + (75 * 10^{-9})} = \frac{3}{5} = 0.6$$

$$Refractive\ index\ (n) = 3.6$$

$$\theta = \sin^{-1}\frac{1}{n} = 16.1276$$

Extraction Efficiency = $\eta_{extraction} = \eta_{internal} * k * \left[1 - \left(\frac{n-1}{n+1}\right)^2\right] * \frac{2\pi(1-\cos\theta)}{4\pi}$

$$\eta_{extraction} = 0.6 * 1 * \left[1 - \left(\frac{3.6-1}{3.6+1}\right)^2\right] * \frac{2\pi(1-\cos(16.1276))}{4\pi} = 0.196$$