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Declaration

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Programme: GTECV - Graduate Training Visitor Program

Module Code: EE506

Assignment Title: Semester Two Solution Assignment 2015

Submission Date: 23 April 2015

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I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Name: Amandeep Kaur

Date: 23 April 2015

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Ques1 The set of equations given below are called rate equations

$$\begin{cases} \frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e} \\ \frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_P} + \beta\frac{N}{\tau_e} \end{cases}$$

What does this set of equations describe? Identify and name all the symbols appearing in Eq. 1.
This set of equations describes the time evolution of both of the carrier number and the photon number.

Sol → The given equations form the two set of the differential equation which describes the time evolution of both carrier number and photon number.
I defines value of current applied to laser
A is gain coefficient
N is Number of carriers in conduction and valence band.
τ_e is carrier lifetime
τ_P is photon lifetime
β is spontaneous emission coupling to mode.
$\frac{N}{\tau_e}$ represent spontaneous emission / recombination of carrier due to spontaneous emission.
$\frac{I}{e}$ represent number of injected carrier per unit time
~~GP represent~~

$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_P} + \frac{\beta N}{\tau_e}$$ describes photon time evolution.

1.2 Use Eq(1) to determine the expression for the current threshold and from list of constants, calculate threshold current. What assumption do you use facilitate calculation

Sol 1.2 → The rate equation should be used in steady state to determine threshold current which means that there is no variation in carrier and photon number ie $\frac{d}{dt} = 0$

As Gain is linear function of carrier then

$$G = A(N-N_0)$$

Now $$\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e} \rightarrow eq(1)$$

$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_P} + \frac{\beta N}{\tau_e}$$

To achieve, threshold current spontaneous emission can be neglected

$GP >> \frac{\beta N}{\tau_e}$. Therefore $\frac{\beta N}{\tau_e}$ canbe neglected.

Put $\frac{dP}{dt} = 0$

$$A(N-N_0)P + \frac{\beta N}{\tau_e} - \frac{P}{\tau_P} = 0$$

$$A(N-N_0)\cancel{P} = \frac{\cancel{P}}{\tau_P}$$

$$A(N-N_0) = \frac{1}{\tau_P}$$

And $$N = N_0 + \frac{1}{A\tau_P}$$

Put value of in eq(1)

$$\frac{I}{e} - \frac{N}{\tau_e} = A(N-N_0)P$$

$$\frac{I}{e} - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_P}\right) = \frac{P}{\tau_P}$$

$$P = \frac{\tau_P}{e}\left[I - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_P}\right)\right]$$

Put $$I_S = \frac{e}{\tau_e}\left[N_0 + \frac{1}{A\tau_P}\right]$$

J_s is threshold current.

Now $J_s = \frac{e}{\tau_e}\left[N_o + \frac{1}{A\tau_p}\right]$

By putting value of constants

$e = 1.6 \times 10^{-19}\ C$

$\tau_e = 2.2\ ns = 2.2 \times 10^{-9}\ s$

$\tau_p = 1.6\ ps = 1.6 \times 10^{-12}\ s$

$N_o = 1.2 \times 10^{8}$

$A = 8 \times 10^{3}\ s^{-1}$

$$J_s = \frac{1.6 \times 10^{-19}\ C}{2.2 \times 10^{-9}\ s}\left[1.2 \times 10^{8} + \frac{1}{8 \times 10^{3}\ s^{-1} \times 1.6 \times 10^{-12}\ s}\right]$$

$J_s = 0.0144\ A$

$J_s = 14.4\ mA$ Ans.

1.3 Determine the expression for the number of the carriers as a function of the bias current set above threshold and below threshold. Comment these result.

Sol → Above the threshold, $A(N - N_o)P >> \frac{BN}{\tau_e}$

So P can be expressed as $P = \tau_p \frac{(J - J_s)}{e}$ as P increases with current and N is clamped to $N_o + \frac{1}{A\tau_p}$ where Gain is equal to losses and carrier number is fixed to corresponding amount of carrier matching the losses. So $N = N_o + \frac{1}{A\tau_p}$

Now Below Threshold, P is negligible Hence $A(N - N_o)P = 0$, Hence $N = \frac{I\tau_e}{e}$. As carrier number increases with current and more carriers are injected in junction.

$N = \frac{I\tau_e}{e}$ Ans

Q2.1 If $\tilde{N}, \tilde{P}$ and $\tilde{I}$ represent steady state quantities, carry out a small signal analysis for rate equation from Ques1, for following perturbations δN, δP and δI around these steady state points. Your solution can be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$ where M is 2X2 vector

Sol ⇒ To carry out small signal analysis, the spontaneous emission should be neglected $\frac{BN}{\tau e} = 0$. The rate equation should be considered at above threshold.

$$\left(\frac{d\tilde{N} + \delta N}{dt}\right) = \frac{\tilde{I} + \delta t}{e} - A(\tilde{N} + \delta N - N_0)(\tilde{P} + \delta P) - \left(\frac{\tilde{N} + \delta\tilde{N}}{\tau e}\right)$$

$$d\frac{(\tilde{P} + \delta P)}{dt} = A(\tilde{N} + \delta N - N_0)(\tilde{P} + \delta P) - \left(\frac{\tilde{P} + \delta P}{\tau P}\right)$$

We neglect second order expression The eqn can be written as

$$\frac{dN}{dt} + \frac{d\delta N}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau e} + \frac{\delta I}{e} - A(\tilde{N} - N_0)\delta P -$$
$$A\tilde{P}\delta N - A\delta N\delta P - \frac{\delta N}{\tau e}$$

$$\frac{dP}{dt} + \frac{d\delta P}{dt} = A(N - N_0)P - \frac{P}{\tau P} + A(N - N_0)\delta P + AP\delta N + A\delta N\delta P$$
$$- \frac{\delta P}{\tau_P}$$

Neglected 2nd Order Term.

$$\frac{d\delta N}{dt} = \frac{\delta I}{e} - A(\tilde{N} - N_0)\delta P - \left(A\tilde{P} + \frac{1}{\tau e}\right)\delta N$$

$$\frac{d(\delta P)}{dt} = A\left((\tilde{N} - N_0) - \frac{1}{\tau P}\right)\delta P + A\tilde{P}\delta N$$

So, Lasing is carried out above threshold ie

$A(\tilde{N} - N_0) = \frac{1}{\tau p}$ then the

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \frac{\delta P}{\tau p} - (A\tilde{P} + \frac{1}{\tau e})\delta N$$

$$\frac{d(\delta P)}{dt} = A\tilde{P}\delta N$$

$$\begin{pmatrix} \frac{d\,\delta N}{dt} \\ \frac{d\,\delta P}{dt} \end{pmatrix} = \begin{bmatrix} -(A\tilde{P} + \frac{1}{\tau e}) & -\frac{1}{\tau p} \\ A\tilde{P} & 0 \end{bmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$

$$M = \begin{bmatrix} -(A\tilde{P} + \frac{1}{\tau e}) & -\frac{1}{\tau p} \\ A\tilde{P} & 0 \end{bmatrix}$$

Q2.2. Identify all the elements of matrix, M.

2. Sol 2.2 → where A is gain coefficient

P is Photon Number

τp is Photon lifetime

τe is carrier lifetime

Q2.3 → If $\delta\tilde{N}$, $\delta\tilde{P}$ and $\delta\tilde{I}$ are expression of first order pertubation of δN, δP and δI in fourier domain, derive the expression for $\delta\tilde{N}$ and $\delta\tilde{P}$ as a function $\delta\tilde{I}$.

Sol → In fourier domain, the elements of matrix above are invariant and this equation can be written as

$$\begin{pmatrix} j\omega\delta\tilde{N} \\ j\omega\delta\tilde{P} \end{pmatrix} = \begin{bmatrix} -(AP + \frac{1}{\tau e}) & -\frac{1}{\tau p} \\ Ap & 0 \end{bmatrix} \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix} + \begin{pmatrix} \frac{\delta\tilde{I}}{e} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = M^{-1} \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

These equation represent an amplitude model of bias current, δI at angular frequency, ω applied to laser which leads to mode of carrier $\delta\tilde{N}$ and photons $\delta\tilde{P}$ at same angular frequency

$$M\begin{pmatrix}\delta\tilde{N}\\ \delta\tilde{P}\end{pmatrix} = \begin{pmatrix}\delta\tilde{J}/e\\ 0\end{pmatrix}$$

$$M = \begin{bmatrix} j\omega + \left(A\bar{P} + \frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\tilde{P} & j\omega \end{bmatrix}$$

$$\begin{pmatrix}\delta N\\ \delta P\end{pmatrix} = M^{-1}\begin{pmatrix}\frac{\delta\tilde{I}}{e}\\ 0\end{pmatrix}$$

$$M^{-1} = \frac{1}{\omega^2 - \frac{A\tilde{P}}{\tau_p} - j\omega\left(\hat{A}P + \frac{1}{\tau_e}\right)}\begin{pmatrix} -j\omega & \\ -A\tilde{P} & -j\omega - \left(A\tilde{P} + \frac{1}{\tau_e}\right)\end{pmatrix}$$

$$\omega_R^2 = \frac{A\tilde{P}}{\tau_p} \quad \text{and} \quad \frac{2}{\tau_P} = A\tilde{P} + \frac{1}{\tau_e}$$

ω_R is relaxation oscillation.

$$\delta\tilde{P} = \left(\frac{-A\tilde{P}}{\left(\omega^2 - \frac{AP}{\tau_P} - j\omega\left(A\tilde{P} + \frac{1}{\tau_e}\right)\right)}\right)\frac{\delta I}{e}$$

$$\frac{\delta P}{\delta J} = \frac{-\tau_P\,\omega_R^2/e}{\omega^2 - \frac{A\tilde{P}}{\tau_P} - j\omega\left(\tilde{A}P + \frac{1}{\tau_e}\right)} \qquad \text{Put } AP = \tau_P\,\omega_R^2$$

$$\delta\tilde{N} = \frac{-j\omega}{\omega^2 - \frac{A\tilde{P}}{\tau_P} - j\omega\left(A\tilde{P} + \frac{1}{\tau_e}\right)}\,\frac{\delta\tilde{J}}{e}$$

$$\frac{\delta\tilde{N}}{\delta J} = \frac{-j\omega/e}{\omega^2 - \frac{\tilde{A}P}{\tau_P} - j\omega\left(A\tilde{P} + \frac{1}{\tau_e}\right)} \qquad \text{Ans}$$

Q3: Consider a photodio with an efficiency $\eta = 0.85$ maximal at wavelength of 1550nm. This photodiode has a bandwidth of 20 GHz with load impedance of 50Ω. It works at temperature of 300°K and an optical power of ~~-3dBm~~ -3dBm launched and its input.

Sol 1 What is Signal to noise ratio of this photodiode?

Sol 3.1 → Here, we know that Signal to noise ratio is given by

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4KT}{R_c}\right)\Delta f}$$

$\eta_e = 0.85$

$\Delta f = 20 \times 10^9$ Hz

$R_c = 50\ \Omega$

$T = 300$ K

$P_s = -3$ dBm

$P(W) = 10^{(P(dB/10))}/1000$

$P_s = 0.0005011$ W

$h = 6.626 \times 10^{-34}$ Js

$e = 1.6021 \times 10^{-19}$ C

$L = 1550 \times 10^{-9}$ m

$c = 3 \times 10^{8}$ m/s

$k = 1.38 \times 10^{-23}$ J/K

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4KT}{R_c}\right)\Delta f}$$

$$= \frac{\left(\frac{0.85 \times 1550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}\right)^2 \cdot (0.0005011)^2}{\left\{\left(2 \times 1.6021 \times 10^{-19}\left(\frac{0.85 \times 1550 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}}\right)0.0005011 + \frac{4 \times 300 \times 1.60 \times 10}{50}\right) \times 20 \times 10^{9}\right.}$$

$\frac{S}{N}$ = 28209.8 W

Now convert Watt into dB

$P(dBm) = 10 \cdot \log_{10}(P(W)/1W)$

$\frac{S}{N}$ = 44.50 dB Ans.

Q 3.2 What is the value of signal to noise ratio if temperature is set at 0°C?

Sol → If the temperature is 0°C then it means 273 K.

Putting T = 273 K in

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P^2 s}{\left(2e\left(\frac{\eta e}{h\nu}\right)Ps + \frac{4KT}{R_C}\right)\Delta f}$$

$\frac{S}{N}$ = 44.77 dB. Ans.

Q 3.3 If the temperature of a photodiode increases, what is the consequence on the signal to noise ratio? Justify your answer?

Sol 3.3 → If the temperature increases, the thermal noise increases which increase the overall noise level. Therefore with increase in noise the Signal to Noise Ratio decreases.

Q. 3.4. If the bandwidth of a photodiode increases, what is the consequence on the signal to noise ra[tio] Justify your answer?

Sol 3.4 As ~~If~~ noise captured by photodiode are independent from the frequency bandwidth of the detector ie white noise. When Δf increases, this means that the photodiode can capture more noise. And, therefore noise level increases then Signal to noise ratio decreases.

Q4. What is value of the internal and external efficiency of the light emitting diode. The radiative and non radiative ~~efficiencies~~ carrier lifetimes are 50ns and 75ns. We assume that there is no default in semiconductor material and contacts are not light absorbing. The refractive index is 3.6 and emission at wavelength of 450nm and bias current of 300mA.

Sol4 → Given radiative carrier lifetime $\tau_r = 50ns$

Non radiative carrier lifetime $\tau_{nr} = 75ns$.

Internal efficiency $$\eta_{int} = \frac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}}$$

$$= \frac{75}{50+75} = \frac{3}{5} = \cdot 60 = 60\%$$

Internal efficiency $= 60\%$

Now for ~~External~~ Extraction efficiency of LED.

Given refractive index of LED $= 3.6$.

$$\eta_{extract} = \frac{2\pi(1 - \cos\theta_c)}{4\pi}$$ As $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ where $n_2 = 1$ and $n_1 = 3.6$.

$$= \frac{1}{2}(1 - \cos\theta_c)$$

$$= 0.019 \text{ or } 0.02$$

Extraction efficiency of LED $= 2\%$ Ans