Appendix A Student Declaration of Academic Integrity

Students may be required to submit work for assessment in a variety of means, for example physical submission or electronic submission through Moodle. In all cases students must make a declaration of academic integrity, either by physically completing such a declaration and submitting it with their assignment or engaging appropriately with the electronic version of the student declaration. Assignments submitted such that the form has not been included, or the electronic equivalent has been circumvented, will not be accepted.

Declaration

Name: SHANE DUGGAN

Student ID Number: 15100715

Programme:

Module Code: EE506

Assignment Title:

Submission Date: 23/4/15

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

I have identified and included the source of all facts, ideas, opinions, viewpoints of others in the assignment references. Direct quotations from books, journal articles, internet sources, module text, or any other source whatsoever are acknowledged and the sources cited are identified in the assignment references.

I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

I have read and understood the DCU library referencing guidelines (available at: http://www.library.dcu.ie/LibraryGuides/DCU Library Guide to Harvard Style of Citing & Referencing/player.html Referencing/player.html accessed 9th January 2013) and/or recommended in the assignment guidelines and/or programme documentation.

By signing this form or by submitting this material online I confirm that this assignment, or any part of it, has not been previously submitted by me or any other person for assessment on this or any other course of study.

By signing this form or by submitting material for assessment online I confirm that I have read and understood DCU Academic Integrity and Plagiarism Policy (available at: http://www4.dcu.ie/registry/examinations/plagiarism.pdf accessed 9th January 2013)

Name: Shane Duggan

Date: 23/4/15

Report of Suspected Plagiarism / Breach of Academic Integrity

1.1 The set of equations describe the evolution of the population of carriers N and photons P with time t.

I = current applied to the laser

e = electron charge

so I/e = injected carriers with time.

A = linear gain coefficient,

when multiplied by $(N-N_0)$, where N_0 = the carrier number when material is transparent, it gives the gain $A(N-N_0)$. As mentioned when $N=N_0$, $A(N-N_0) \to 0$

Then $A(N-N_0)P$ represents the number of carriers gone into producing photons, this is stimulated emission

τ_e is the lifetime of the carriers, so N/τ_e represents the carriers that decay to the valence band, this is spontaneous emission.

τ_p is the photon lifetime so that P/τ_p represents the photons that have escaped, the cavity losses.

β = the spontaneous emission coupled to the mode. Then $\beta N/\tau_e$ gives the number of photons generated by spontaneous emission that are confined to the cavity direction

1.2 Look at steady state where the carrier and photon numbers are not changing $\Rightarrow d/dt \to 0$

At threshold the stimulated emission dominates so neglect the spontaneous emission $\Rightarrow \beta N/\tau_e \to 0$ as $A(N-N_0)P >> \beta N/\tau_e$.

Then the rate equation for P becomes

$$0 = A(N-N_0)P - P/\tau_p$$

$$\Rightarrow A(N-N_0) = 1/\tau_p \qquad \text{Gain = losses}$$

$$\Rightarrow N = N_0 + \frac{1}{A\tau_p}$$

The rate equation for N is then:

$$0 = I/e - P/\tau_p - N/\tau_e$$
$$= I/e - P/\tau_p - \frac{N_0 + 1/A\tau_p}{\tau_e}$$

$$\Rightarrow \quad P = \frac{\tau_p I}{e} - \frac{\tau_p}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$
$$= \frac{\tau_p}{e}\left[I - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)\right]$$
$$= \frac{\tau_p}{e}\left(I - I_{th}\right)$$

so the threshold current is $I_{th} = e/\tau_e \left(N_0 + 1/A\tau_p\right)$

Using
$$e = 1.6 \times 10^{-19}\ C$$
$$\tau_e = 2.2\,ns = 2.2 \times 10^{-9}\ s$$
$$\tau_p = 1.6\ ps = 1.6 \times 10^{-12}\ s$$
$$A = 8 \times 10^{3}\ s^{-1}$$
$$N_0 = 1.2 \times 10^{8}$$

$$\Rightarrow I_{th} = \frac{1.6 \times 10^{-19}}{2.2 \times 10^{-9}}\left(1.2 \times 10^{8} + \frac{1}{8 \times 10^{3} \times 1.6 \times 10^{-12}}\right)$$

$$= 0.0144\ A$$

$$= 14.4\ mA$$

1.3 Above threshold $N = N_0 + 1/A\tau_p$ from Q1.2.
This is a fixed value, corresponding to the gain = losses

Below threshold spontaneous emission now dominates
$$\Rightarrow \quad A(N - N_0)P << \beta N/\tau_e$$
So rate equation for P becomes
$$0 = \beta N/\tau_e - P/\tau_p$$
while rate equation for N becomes
$$0 = I/e - N/\tau_e$$

because if $\beta N/\tau_e \gg A(N-N_0)P$
then $N/\tau_e \gg A(N-N_0)P$ too.

Thus $N = \frac{\tau_e I}{e}$
so that the carrier number is linearly proportional to the current and will increase with current until threshold

2.1 Steady state and set above threshold current so $\beta N/\tau_e \ll A(N-N_0)P$. so in the rate equation drop $\beta N/\tau_e$.
Inserting $\bar{N}+\delta N$, $\bar{P}+\delta P$ and $\bar{I}+\delta I$ into the rate eq^{n}s gives:

$$\frac{d}{dt}(\bar{N}+\delta N) = \frac{\bar{I}+\delta I}{e} - A(\bar{N}+\delta N-N_0)(\bar{P}+\delta P) - \frac{(\bar{N}+\delta N)}{\tau_e}$$
$$\frac{d}{dt}(\bar{P}+\delta P) = A(\bar{N}+\delta N-N_0)(\bar{P}+\delta P) - \frac{\bar{P}+\delta P}{\tau_p}$$

$$\Rightarrow \frac{d\bar{N}}{dt} + \frac{d\delta N}{dt} = \frac{\bar{I}}{e} + \frac{\delta I}{e} - A(\bar{N}-N_0)\bar{P} - A(\bar{N}-N_0)\delta P - A\delta N\bar{P} - A\delta N\delta P - \frac{\bar{N}}{\tau_e} - \frac{\delta N}{\tau_e}$$
$$\frac{d\bar{P}}{dt} + \frac{d\delta P}{dt} = A(\bar{N}-N_0)\bar{P} + A(\bar{N}-N_0)\delta P + A\delta N\bar{P} + A\delta N\delta P - \frac{\bar{P}}{\tau_p} - \frac{\delta P}{\tau_p}$$

Now using rate equations for the steady state quantities

$$\frac{d\bar{N}}{dt} = \frac{\bar{I}}{e} - A(\bar{N}-N_0)\bar{P} - \bar{N}/\tau_e = 0$$
and $$\frac{d\bar{P}}{dt} = A(\bar{N}-N_0)\bar{P} - \bar{P}/\tau_p = 0 \qquad (*)$$

$$\Rightarrow \frac{d\delta N}{dt} = \frac{\delta I}{e} - A(\bar{N}-N_0)\delta P - A\delta N\bar{P} - A\delta N\delta P - \delta N/\tau_e$$
$$\frac{d\delta P}{dt} = A(\bar{N}-N_0)\delta P + A\delta N\bar{P} + A\delta N\delta P - \delta P/\tau_p$$

and limit analysis to first order so take $O(\delta^2)\to 0$, using also $A(\bar{N}-N_0) = 1/\tau_p$ from $(*)$

$$\Rightarrow \frac{d\delta N}{dt} = \frac{\delta I}{e} - \frac{\delta P}{\tau_p} - A\delta N\bar{P} - \delta N/\tau_e$$
$$\frac{d\delta P}{dt} = \cancel{\delta P/\tau_p} + A\delta N\bar{P} - \cancel{\delta P/\tau_p}$$

which can be written in matrix form as

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -A\bar{P} - 1/\tau_e & -1/\tau_p \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$

$$= M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

so that $M = \begin{pmatrix} -A\bar{P} - 1/\tau_e & -1/\tau_p \\ A\bar{P} & 0 \end{pmatrix}$

2.2 $M(0,0) = -A\bar{P} - 1/\tau_e$ represents the losses in the perturbation carrier number due to stimulated and spontaneous emission, $-A\bar{P}$ and $-1/\tau_e$ respectively.

$M(0,1) = -1/\tau_p$ represents the decay of the perturbation photon number due to the photon lifetime

$M(1,0) = A\bar{P}$ represents the generation of photons due to the gain

$M(1,1) = 0$ indicates that the behavior of the perturbation photons δP doesn't depend on itself directly. It depends on δN which in turn does depend on δP.

2.3 Take Fourier Transform of solution from 2.1

$FT\{\delta N\} = \delta\tilde{N}$ $\quad FT\{\frac{d\delta N}{dt}\} = j\omega\delta\tilde{N}$

$FT\{\delta P\} = \delta\tilde{P}$ $\quad FT\{\frac{d\delta P}{dt}\} = j\omega\delta\tilde{P}$

$FT\{\delta I\} = \delta\tilde{I}$

$$\Rightarrow \begin{pmatrix} j\omega\delta\tilde{N} \\ j\omega\delta\tilde{P} \end{pmatrix} = \begin{pmatrix} -A\bar{P} - 1/\tau_e & -1/\tau_p \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix} + \begin{pmatrix} \delta\tilde{I}/e \\ 0 \end{pmatrix}$$

or $\begin{pmatrix} j\omega & 0 \\ 0 & j\omega \end{pmatrix}\begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{p} \end{pmatrix} = \begin{pmatrix} -A\bar{p} - 1/\tau_e & -1/\tau_p \\ A\bar{p} & 0 \end{pmatrix}\begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{p} \end{pmatrix} + \begin{pmatrix} \delta \bar{I}/e \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} j\omega + A\bar{p} + 1/\tau_e & 1/\tau_p \\ -A\bar{p} & j\omega \end{pmatrix}\begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{p} \end{pmatrix} = \begin{pmatrix} \delta\bar{I}/e \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{p} \end{pmatrix} = \begin{pmatrix} j\omega + A\bar{p} + 1/\tau_e & 1/\tau_p \\ -A\bar{p} & j\omega \end{pmatrix}^{-1}\begin{pmatrix} \delta\bar{I}/e \\ 0 \end{pmatrix}$

$$= \frac{1}{(j\omega + A\bar{p} + 1/\tau_e)(j\omega) + A\bar{p}/\tau_p}\begin{pmatrix} j\omega & -1/\tau_p \\ A\bar{p} & j\omega + A\bar{p} + 1/\tau_e \end{pmatrix}\begin{pmatrix} \delta\bar{I}/e \\ 0 \end{pmatrix}$$

$$= \frac{1}{-\omega^2 + j\omega A\bar{p} + j\,\omega/\tau_e + A\bar{p}/\tau_p}\begin{pmatrix} j\omega & -1/\tau_p \\ A\bar{p} & j\omega + A\bar{p} + 1/\tau_e \end{pmatrix}\begin{pmatrix} \delta\bar{I}/e \\ 0 \end{pmatrix}$$

Write $\omega_R^2 = A\bar{p}/\tau_p$ and $2/\tau_R = A\bar{p} + 1/\tau_e$
so that

$$\begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{p} \end{pmatrix} = \frac{1}{(\omega^2 - \omega_R^2) - j\omega\, 2/\tau_R}\begin{pmatrix} -j\omega & 1/\tau_p \\ -A\bar{p} & -j\omega - A\bar{p} - 1/\tau_e \end{pmatrix}\begin{pmatrix} \delta\bar{I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow \delta\tilde{N} = \frac{-j\omega\, \delta\bar{I}/e}{(\omega^2 - \omega_R^2) - 2j\,\omega/\tau_R}$$

$$\delta\tilde{p} = \frac{-A\bar{p}\, \delta\bar{I}/e}{(\omega^2 - \omega_R^2) - 2j\,\omega/\tau_R}$$

3.1 $$S/N = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_c}\right)\Delta f}$$

$\eta = 0.85$
$e = 1.6 \times 10^{-19}$ C
$h = 6.62 \times 10^{-34}$ Js
$\nu = c/\lambda = \frac{3\times10^8}{1550\times10^{-9}} = 1.9355 \times 10^{14}$ Hz
$P_s = -3$ dBm
$\Rightarrow 10\log(P_s/1\,\text{mW}) = -3$
$\Rightarrow P_s = 10^{-3/10}$ mW
$= 0.5$ mW
$= 0.0005$ W
$k = 1.38 \times 10^{-23}$ JK^{-1}
$R_c = 50\ \Omega$
$\Delta f = 20$ GHz $= 20 \times 10^9$ Hz

For $T = 300$ K

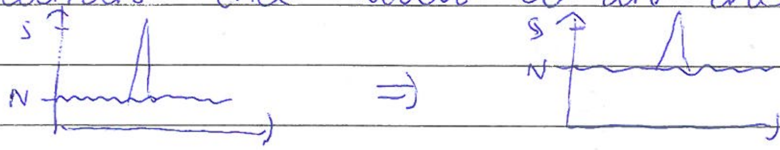
$\Rightarrow S/N = 28107.6$
$= 44.5$ dB (using $10\log(S/N)$)

3.2 At $0°C = 273$ K

$S/N = 29885.6$
$= 44.8$ dB

3.3 With increasing T saw in 3.1 and 3.2 that the signal-to-noise ratio S/N decreased.
This is because $S/N \approx 1/T$ so that if T↑ S/N ↓

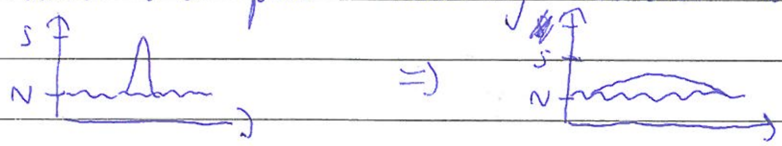
Physically this corresponds to thermal excitation of carriers and with it an increase in the noise



3.4 If the bandwidth of increases then the signal-to-noise ratio decreases as

$$S/N \propto 1/\Delta f$$

Physically this occurs because the signal broadens with the peak lowering toward the noise level, as well as the broadened signal capturing more noise



4.1 Internal quantum efficiency $\eta_{int} = \frac{1/\tau_R}{1/\tau_R + 1/\tau_{NR}} = \frac{1}{1+\frac{\tau_R}{\tau_{NR}}}$

For $\tau_R = 50ns$, $\tau_{NR} = 75ns$

$$\Rightarrow \quad \eta_{int} = \frac{1}{1+50/75}$$

$$= 0.6$$

For the extraction efficiency:

Some photons generated internally will be reflected from the interface due to the index change. The transmittance is $T = 1-R$ for the reflectance $R = \left(\frac{n-1}{n+1}\right)^2$

$$\Rightarrow \quad T = 1 - \left(\frac{n-1}{n+1}\right)^2$$

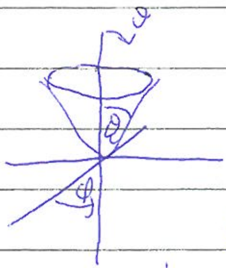
$$= \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{(n+1+n-1)(n+1-n+1)}{(n+1)^2}$$

$$= \frac{4n}{(n+1)^2}$$

In addition some light will be totally internally reflected. Transmission will only occur within a

cone defined by the critical angle θ_c.
From Snell's law this angle is

$$n \sin\theta_c = 1$$
$$\Rightarrow \sin\theta_c = 1/n$$



Both cones are rotated 2π in ϕ-direction, it is the fraction in the θ-direction that must be found:

$$\frac{\int_0^{\theta_c} \sin\theta \, d\theta}{\int_0^{\pi} \sin\theta \, d\theta} = \frac{-\cos\theta \big|_0^{\theta_c}}{-\cos\theta \big|_0^{\pi}} = \frac{-\cos\theta_c + 1}{1+1}$$

$$= \frac{1-\cos\theta_c}{2}$$

Can approximate $\sin\theta_c \sim \theta_c$
$\cos\theta_c \sim 1 + \theta_c^2/2$

or else put in fully.

$$\int_0 \frac{1-\cos\theta_c}{2} \rightarrow \frac{1-(1+\theta_c^2/2)}{2} = \theta_c^2/4 = \frac{1}{4n^2}$$

Together with the transmittance: $\frac{4n}{(n+1)^2}\frac{1}{4n^2} = \frac{1}{(n+1)^2 n}$

For $n = 3.6$ this equals 0.013

The extraction efficiency then is $\eta_{int} \cdot \frac{1}{(n+1)^2 n}$
$= (0.6)(0.013)$
$= 0.000788$

when it has been taken there is no absorption as there are no defaults in the semiconductor and the contacts aren't light absorbing.