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Declaration

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Programme: PHD ~~PHD~~

Module Code: EES06

Assignment Title: FUNDAMENTALS OF PHOTOPIC DEVICES

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Name: Niamh Kavanagh

Date: 23/4/15

Report of Suspected Plagiarism / Breach of Academic Integrity

Question 1

1.1 Rate Equations

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + B\frac{N}{\tau_e}$$

What does this set of eqns describe?

dN/dt describes the carrier time evolution and dP/dt describes the photon time evolution.

Identify + name all the symbols.

t : time

N : number of carriers

I : current

e : electron charge

A : gain coefficient

N_0 : carrier number at transparency
(when light input = light output)

P : number of photons

τ_e : carrier lifetime

τ_p : photon lifetime

B : spontaneous emission coupled to the mode.

1.2 Use these eqns. to determine the expression for the threshold current.

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e} \quad (1)$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + \frac{\beta N}{\tau_e} \quad (2)$$

Use steady state $\Rightarrow d/dt = 0$.

At (and above) the threshold stimulated emission becomes dominant. \Rightarrow can neglect spontaneous emission terms $\frac{N}{\tau_e}$ and $\frac{\beta N}{\tau_e}$

$$\Rightarrow \frac{I}{e} = A(N - N_0)P \quad (3)$$

$$\frac{I}{\tau_p} = A(N - N_0) \quad (4)$$

Definition of threshold: gain = losses

$$(4) \quad \frac{I}{\tau_p} = A(N - N_0) \quad \frac{I}{\tau_p} = AN - N_0A$$

$$AN = \frac{I}{\tau_p} + N_0A \quad \boxed{N = N_0 + \frac{I}{A\tau_p}} \quad (5)$$

This is the number of carriers at the threshold

Fill eqn. (5) into eqn. (1)

$$0 = \frac{I}{e} - A(N - N_0)P - \frac{b}{\tau_e}$$

where $N = N_0 + \frac{1}{A\tau_p}$

$$\frac{I}{e} - A\left(N_0 + \frac{1}{A\tau_p} - N_0\right)P - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\frac{I}{e} - \frac{P}{\tau_p} - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right) = 0$$

From lec. 6, slide 20:

$$P = \frac{I_p}{e} (1 - I_{th})$$

$$\frac{eP}{\tau_p} = \frac{I}{e} - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\frac{eP}{\tau_p} = I - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\Rightarrow P = \frac{I_p}{e} \left[I - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right) \right]$$

I_{th}

$$\Rightarrow I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

1.2

$$I_{th} = \frac{e}{\tau_e} \left(N_0 + \frac{1}{A \tau_p} \right)$$

$$e = 1.6 \times 10^{-19} \text{ C} \quad \tau_e = 2.2 \times 10^{-9} \text{ s}$$

$$N_0 = 1.2 \times 10^8 \quad A = 8 \times 10^3 \text{ s}^{-1}$$

$$\tau_p = 1.6 \times 10^{-12} \text{ s}$$

$$\Rightarrow I_{th} = 0.0144 \text{ A}$$

$$I_{th} = 14.43 \text{ mA}$$

What assumption do you use to facilitate this calculation?

Assumptions:

- steady state
- at threshold, stimulated > spontaneous
- $P = \frac{I_p}{e} (I - I_{th})$

eqn. of a line: $y = mx + b$

$$P \propto \frac{I_p}{e} (I - I_{th}) \Rightarrow \frac{I_p}{e} = \text{slope (above threshold)}$$

1.3 Determine the expression for the number of carriers as a function of the bias current set above and below the threshold. Comment.

Above the threshold, the carrier number is clamped (similar to the overflowing bucket analogy, lecture 6).

$$\Rightarrow \text{Above threshold: } N = N_0 + \frac{1}{A \tau_p}$$

Below the threshold:

spontaneous emission > stimulated

$$\Rightarrow \frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_c}$$

Steady State

$$\Rightarrow \frac{I}{e} = \frac{N}{\tau_e} \Rightarrow N = \frac{\tau_e I}{e}$$

Below threshold: $N = \frac{\tau_e I}{e}$

carrier number increases linearly with increasing current.

Question 2

2.1 If \bar{N} , \bar{P} and \bar{I} represent steady state quantities, carry out a small signal analysis for the rate equation from Q.1 for the following perturbations δN , δP and δI around these steady state points.

Solution must be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I \\ 0 \end{pmatrix}$$

lec 6. Without modulation:
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$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + \frac{B N}{\tau_e}$$

Small signal analysis:

Bias current set above threshold

\Rightarrow drop spontaneous emission $\left(\frac{B N}{\tau_e}\right)$

$$I = \bar{I} + \delta I \quad N = \bar{N} + \delta N \quad P = \bar{P} + \delta P$$

$$\frac{d(\bar{N} + \delta N)}{dt} = \frac{\bar{I} + \delta I}{e} - A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{(\bar{N} + \delta N)}{\tau_e}$$

$$\frac{d(\bar{P} + \delta P)}{dt} = A(\bar{N} + \delta N - N_0)(\bar{P} + \delta P) - \frac{(\bar{P} + \delta P)}{\tau_p}$$

Multiplying out

$$\cancel{\frac{d\bar{N}}{dt}} + \frac{d\delta N}{dt} = \frac{\bar{I}}{e} - A(\bar{N} - N_0)\bar{P} - \frac{\bar{N}}{\tau_e} + \frac{\delta I}{e} - A(\bar{N} - N_0)\delta P - A\bar{P}\delta N - A\delta N\delta P - \frac{\delta N}{\tau_e}$$

$$\cancel{\frac{d\bar{P}}{dt}} + \frac{d\delta P}{dt} = A(\bar{N} - N_0)\bar{P} - \frac{\bar{P}}{\tau_p} + A(\bar{N} - N_0)\delta P + A\bar{P}\delta N + A\delta N\delta P - \delta P/\tau_p$$

Use: $\frac{d\bar{N}}{dt} = \frac{\bar{I}}{e} - A(\bar{N} - N_0)\bar{P} - \frac{\bar{N}}{\tau_e}$

$$\frac{d\delta P}{dt} = A(\bar{N} - N_0)\delta P - \frac{\delta P}{\tau_p}$$

and neglect 2nd order terms ($\delta N\delta P = 0$)

$$\Rightarrow \frac{d\delta N}{dt} = \frac{\delta I}{e} - \underline{A(CN - N_0)} \delta P - \underline{A_P} \delta N - \frac{\delta N}{\tau_e}$$

$$\frac{d\delta P}{dt} = + \underline{A(CN - N_0)} \delta P + \underline{A_P} \delta N - \frac{\delta P}{\tau_p}$$

Above threshold, lasing condition: $A(CN - N_0) = 1/\tau_p$

$$\Rightarrow \frac{d\delta P}{dt} = - \frac{\delta P}{\tau_p} - \underline{A_P} \delta N - \frac{\delta N}{\tau_e} + \frac{\delta I}{e}$$

$$\frac{d\delta P}{dt} = \cancel{\frac{\delta P}{\tau_p}} + \underline{A_P} \delta N - \cancel{\frac{\delta P}{\tau_p}}$$

$$\Rightarrow \frac{d\delta N}{dt} = - \frac{1}{\tau_p} \delta P - \left(\underline{A_P} + \frac{1}{\tau_e} \right) \delta N + \frac{\delta I}{e}$$

$$\frac{d\delta P}{dt} = 0 \delta P + \underline{A_P} \delta N + 0$$

solution

$$\Rightarrow \frac{d}{dt} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = \underbrace{\begin{pmatrix} -(A_P + 1/\tau_e) & -1/\tau_p \\ A_P & 0 \end{pmatrix}}_M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

$$\Rightarrow \text{if } \frac{d}{dt} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

$$\Rightarrow M = \begin{pmatrix} -(A_P + 1/\tau_e) & -1/\tau_p \\ A_P & 0 \end{pmatrix}$$

2.2 Identify all elements of Matrix M

2.3 If $\tilde{\delta N}$, $\tilde{\delta P}$ and $\tilde{\delta I}$ are the expression of the 1st order perturbation of δN , δP and δI in the Fourier domain, derive the expressions for $\tilde{\delta N}$ and $\tilde{\delta P}$ as a f. of $\tilde{\delta I}$.

Fourier Transform: $\delta x(t) \rightarrow \delta x(\omega)$
 $\frac{d}{dt} \rightarrow j\omega$

$$\frac{d}{dt} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = \begin{pmatrix} -CAP + 1/\tau_e & -1/\tau_p \\ AP & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/\epsilon \\ 0 \end{pmatrix}$$

After Fourier Transform...

$$j\omega \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = \begin{pmatrix} -CAP + 1/\tau_e & -1/\tau_p \\ AP & 0 \end{pmatrix} \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} + \begin{pmatrix} \tilde{\delta I}/\epsilon \\ 0 \end{pmatrix}$$

Want to get expressions for $\tilde{\delta N}$ and $\tilde{\delta P}$

Multiply $j\omega$ by identity to get 2x2

$$\Rightarrow j\omega \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} - \begin{pmatrix} -CAP + 1/\tau_e & -1/\tau_p \\ AP & 0 \end{pmatrix} \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = \begin{pmatrix} \tilde{\delta I}/\epsilon \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} \left[\begin{pmatrix} j\omega & 0 \\ 0 & j\omega \end{pmatrix} + \begin{pmatrix} CAP + 1/\tau_e & 1/\tau_p \\ -AP & 0 \end{pmatrix} \right] = \begin{pmatrix} \tilde{\delta I}/\epsilon \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} \begin{bmatrix} j\omega + AP + 1/\tau_e & 1/\tau_p \\ -AP & j\omega \end{bmatrix} = \begin{pmatrix} \tilde{\delta I}/\epsilon \\ 0 \end{pmatrix}$$

$$\text{Now } M = \begin{pmatrix} j\omega + AP + 1/\tau_e & 1/\tau_p \\ -AP & j\omega \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} \bar{I}_N \\ \bar{I}_P \end{pmatrix} = \begin{pmatrix} \bar{I}/c \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \bar{I}_N \\ \bar{I}_P \end{pmatrix} = M^{-1} \begin{pmatrix} \bar{I}/c \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{Then } M^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\begin{aligned} a &= j\Omega + AP + 1/\tau_e & b &= 1/\tau_p \\ c &= -AP & d &= j\Omega \end{aligned}$$

$$\Rightarrow M^{-1} = \frac{1}{(j\Omega + AP + 1/\tau_e)(j\Omega) - (1/\tau_p)(-AP)} \begin{pmatrix} j\Omega & -1/\tau_p \\ AP & j\Omega + AP + 1/\tau_e \end{pmatrix}$$

$$\begin{aligned} \det &= j^2 \Omega^2 + j\Omega AP + \frac{j\Omega}{\tau_e} + \frac{AP}{\tau_p} \\ &= -\Omega^2 + \frac{AP}{\tau_p} + j\Omega \left(AP + \frac{1}{\tau_e} \right) \end{aligned}$$

$$\Rightarrow M^{-1} = \frac{1}{-\Omega^2 + \frac{AP}{\tau_p} + j\Omega \left(AP + \frac{1}{\tau_e} \right)} \begin{pmatrix} j\Omega & -1/\tau_p \\ AP & j\Omega + AP + \frac{1}{\tau_e} \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$x_1 = ay_1 + by_2 \quad x_2 = cy_1 + dy_2$$

$$\Rightarrow \tilde{\delta}_N = \frac{j\Omega}{-\Omega^2 + \frac{AP}{T_p} + j\Omega \left(AP + \frac{1}{T_e} \right)} \left(\frac{\tilde{\delta}I}{e} \right)$$

$$\tilde{\delta}_P = \frac{AP}{-\Omega^2 + \frac{AP}{T_p} + j\Omega \left(AP + \frac{1}{T_e} \right)} \left(\frac{\tilde{\delta}I}{e} \right)$$

$$\omega_r = \omega_r + \Omega^2 \dots$$

$$\tilde{\delta}_N = \frac{-j\Omega}{\Omega^2 - \frac{AP}{T_p} - j\Omega \left(AP + \frac{1}{T_e} \right)} \left(\frac{\tilde{\delta}I}{e} \right)$$

$$\tilde{\delta}_P = \frac{-AP}{\Omega^2 - \frac{AP}{T_p} - j\Omega \left(AP + \frac{1}{T_e} \right)} \left(\frac{\tilde{\delta}I}{e} \right)$$

From lecture 7:

$$\omega_r^2 = \frac{AP}{T_p} \quad \frac{2}{T_r} = AP + \frac{1}{T_e}$$

$$\Rightarrow \bar{\delta}_N = \frac{-j\Omega}{\Omega^2 - \omega_r^2 - j\frac{\Omega}{T_r}} \left(\frac{\bar{\delta}_I}{e} \right)$$

$$\bar{\delta}_P = \frac{-\textcircled{AP} \xrightarrow{\omega_r^2 T_P}}{\Omega^2 - \omega_r^2 - j\frac{\Omega}{T_r}} \left(\frac{\bar{\delta}_I}{e} \right)$$

$$\Rightarrow \frac{\bar{\delta}_N}{\bar{\delta}_I} = \frac{-j\Omega}{e \left(\Omega^2 - \omega_r^2 - \frac{2j\Omega}{T_r} \right)}$$

$$\frac{\bar{\delta}_P}{\bar{\delta}_I} = \frac{-\omega_r^2 T_P}{e \left(\Omega^2 - \omega_r^2 - \frac{2j\Omega}{T_r} \right)}$$

Q3 Consider a photodiode with efficiency $\eta = 0.85$ Max at $\lambda = 1550 \text{ nm}$. This PD has BW of 20 GHz with load impedance 50Ω . $T_{\text{temp}} = 300 \text{ K}$ and optical power = -30 dBm .

Hint:
$$\frac{S}{N} = \frac{\left(\frac{q\eta}{h\nu}\right)^2 P_s^2}{\left(2e \left(\frac{q\eta}{h\nu}\right) P_s + 4 \frac{kT}{R_L}\right) \Delta f}$$

3.1

For calculations, see overleaf (done in Mathematica)

$$\text{SNR} = 44.5 \text{ dB}$$

3.2 For $T = 0^\circ \text{C}$ $\text{SNR} = 44.77 \text{ dB}$
 $\Rightarrow T = 273 \text{ K}$

3.3 If the T_{temp} increases, what is the consequence to SNR?

As the T_{temp} increases, the thermal noise increases. Thus the noise level increases and the SNR decreases.

ClearAll;

In[3]:= h = 6.62 * 10^-34; n = 0.85; PdBm = -3;

T1 = 300; Rc = 50; deltaf = 20 * 10^9; c = 3 * 10^8;

e = 1.6 * 10^-19; k = 1.38 * 10^-23; L = 1550 * 10^-9;

Defining constants

$$\begin{aligned} c &= f * L \\ f &= c / L \\ 1/f &= L / c \end{aligned}$$

getting freq. in terms of wavelength (L) and speed of light (c).

In[4]:= PW = 10^(PdBm/10)/1000 → converting dBm to Watts.

Out[4]= $\frac{1}{1000 \times 10^{3/10}}$

In[5]:= N[%, 10]

Out[5]= 0.0005011872336 → -3dBm = 0.0005 Watts.

In[10]=

3.1

$$\frac{\left(\frac{n * e}{h * c / L}\right)^2 * (PW)^2}{\left(2 * e * \left(\frac{n * e}{h * c / L}\right) * (PW) + \frac{4 * k * T1}{Rc}\right) * (\text{deltaf})}$$

Out[10]= 28218.9

In[11]:= PdB = 10 * Log10[%] → converting to dB

Out[11]= 44.5054 dB

In[8]=

For T=300k, S/P = 44.5 dB

T2 = 273;

3.2

$$\frac{\left(\frac{n * e}{h * c / L}\right)^2 * (PW)^2}{\left(2 * e * \left(\frac{n * e}{h * c / L}\right) * (PW) + \frac{4 * k * T2}{Rc}\right) * (\text{deltaf})}$$

Out[12]= 30002.5

In[13]:= PdB = 10 * Log10[%]

Out[13]= 44.7716 dB

For T=273k, S/P = 44.77 dB

3.4. If the BW increases, what is the consequence for S/P? When the BW increases, this means that the PD can capture more noise. Thus the noise level increases and the S/P decreases.

Q.1 What is the internal and external quantum efficiency of the LED?

Radiative lifetime = 30 ns $\rightarrow \tau_r$
 Non-radiative lifetime = 75 ns $\rightarrow \tau_{nr}$
 refractive index = 3.6 $\rightarrow n$
 emission max. @ $\lambda = 450 \text{ nm} \rightarrow \lambda$
 bias current = 300 mA $\rightarrow I$

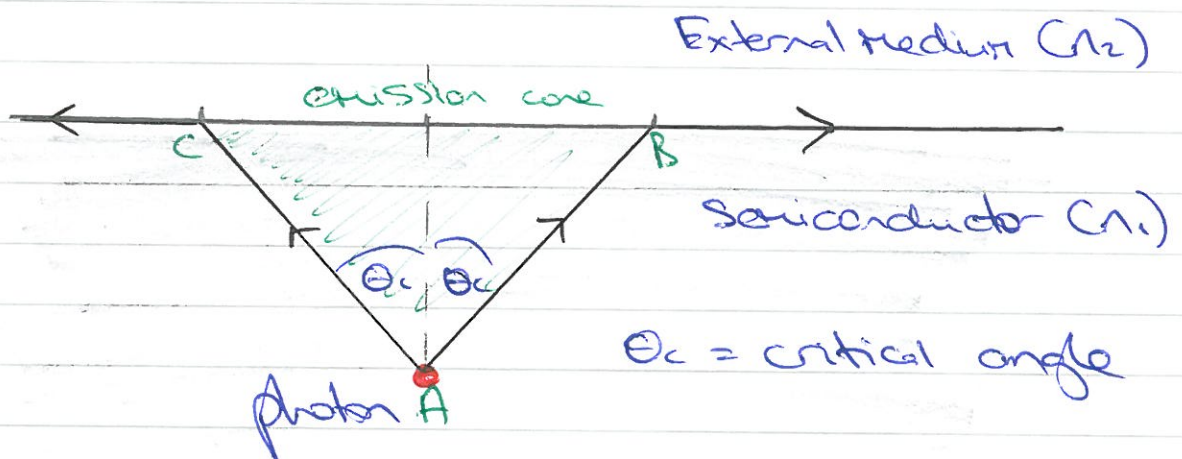
assume: no defect in S.C. material and contacts are not light absorbing.

Internal quantum efficiency:

$$\eta_{int} = \frac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}} = \frac{3}{5} = 0.6$$

$$\eta_{int} = 0.6$$

For external efficiency, need to consider solid angle.



Reference: Fiber optics, Prof. R.K. Shevgaonkar, Dept. of Electronic Engineering, Indian I.T. Lecture 14 notes, Optical Sources.

The quantity Γ_{ext} can be defined as the ratio of the solid angle subtended at A by the emission cone to the total solid angle through which "N" photons have equal probability to travel.

Since the generated photons have equal probabilities to travel along all directions at A, the latter solid angle corresponds to 4π .

$$\Rightarrow \Gamma_{\text{ext}} = \frac{2\pi \int_0^{\theta_c} \sin\theta d\theta}{4\pi}$$

$$\Gamma_{\text{ext}} = \frac{1}{2} (1 - \cos\theta_c)$$

Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$
At $\theta_1 = \theta_c$, $\theta_2 = 90^\circ \Rightarrow \sin\theta_2 = 1$

$$\Rightarrow \sin\theta_c = \frac{n_2}{n_1} \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1}{3.6}\right) \quad n_2 = \text{air} = 1$$
$$n_1 = 3.6$$

$$\Rightarrow \theta_c = 16.127^\circ$$

$$\Rightarrow \Gamma_{\text{ext}} = 0.019$$

$$\Gamma_{\text{ext}} \sim 2\%$$