## Appendix A Student Declaration of Academic Integrity

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**Declaration**

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Programme: PHD ~~[illegible]~~

Module Code: EE506

Assignment Title: FUNDAMENTALS OF PHOTONIC DEVICES

Submission Date: 24/4/15 SEMESTER TWO ASSIGNMENT

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

I have identified and included the source of all facts, ideas, opinions, viewpoints of others in the assignment references. Direct quotations from books, journal articles, internet sources, module text, or any other source whatsoever are acknowledged and the sources cited are identified in the assignment references.

I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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By signing this form or by submitting this material online I confirm that this assignment, or any part of it, has not been previously submitted by me or any other person for assessment on this or any other course of study.

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Name: Niamh Kavanagh

Date: 23/4/15

**Report of Suspected Plagiarism / Breach of Academic Integrity**

Question 1

1.1 Rate Equations

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + B\frac{N}{\tau_e}$$

What does this set of eqns. describe?

$dN/dt$ describes the carrier time evolution and $dP/dt$ describes the photon time evolution.

Identify + name all the symbols.

$t$: time
$N$: number of carriers
$I$: current
$e$: electron charge
$A$: gain coefficient
$N_0$: carrier number at transparency (when light input = light output)
$P$: number of photons
$\tau_e$: carrier lifetime
$\tau_p$: photon lifetime
$B$: spontaneous emission coupled to the mode.

1.2 Use these eqns. to determine the expression for the threshold current.

$$\cancel{\frac{dN}{dt}}^{0} = \frac{I}{e} - A(N-N_0)P - \cancel{\frac{N}{\tau_e}} \quad (1)$$

$$\cancel{\frac{dP}{dt}}^{0} = A(N-N_0)\cancel{P} - \frac{\cancel{P}}{\tau_p} + \cancel{\frac{\beta N}{\tau_e}} \quad (2)$$

Use steady state $\Rightarrow$ $d/dt = 0$.

At (and above) the threshold stimulated emission becomes dominant.
$\Rightarrow$ can neglect spontaneous emission terms $\frac{N}{\tau_e}$ and $\beta\frac{N}{\tau_e}$

$$\Rightarrow \quad \frac{I}{e} = A(N-N_0)P \quad (3)$$

$$\frac{1}{\tau_p} = A(N-N_0) \quad (4)$$

Definition of threshold: gain = losses

(4) $\frac{1}{\tau_p} = A(N-N_0)$ $\quad \frac{1}{\tau_p} = AN - N_0A$

$AN = \frac{1}{\tau_p} + N_0A$ $\quad \boxed{N = N_0 + \frac{1}{A\tau_p}}$ (5)

This is the number of carriers at the threshold

Fill eqn. (5) into eqn. (1)

$$0 = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e}$$

where $N = N_0 + \frac{1}{A\tau_p}$

$$\frac{I}{e} - \cancel{A}\left(\cancel{N_0} + \frac{1}{\cancel{A}\tau_p} - \cancel{N_0}\right)P - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\frac{I}{e} - \frac{P}{\tau_p} - \frac{1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right) = 0$$

From lec. 6, slide 20: $\boxed{P = \frac{\tau_p}{e}\left(I - I_{th}\right)}$

$$e\frac{P}{\tau_p} = \frac{I}{\cancel{e}} - \frac{e1}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\frac{eP}{\tau_p} = I - \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$\Rightarrow P = \frac{\tau_p}{e}\left[I - \underbrace{\frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)}_{I_{th}}\right]$$

$$\Rightarrow \boxed{I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)}$$

What assumption do you use to facilitate this calculation?

Assumptions:

- steady state
- at threshold, stimulated > spontaneous
- $P = \frac{\tau_p}{e}(I - I_{th})$

eqn. of a line: $y = mx + b$

$\cancel{P \approx \frac{\tau_p I}{e}}$ $\Rightarrow$ $\frac{\tau_p}{e}$ = slope (above threshold)

1.3 Determine the expression for the number of carriers as a function of the bias current set above and below the threshold. Comment.

Above the threshold, the carrier number is clamped (similar to the overflowing bucket analogy, lecture 6).

$\Rightarrow$ $\boxed{\text{Above threshold: } N = N_0 + \frac{1}{A\tau_p}}$

Below the threshold:
spontaneous emission > stimulated

$\Rightarrow$ $\underbrace{\cancel{\frac{dN}{dt}}}_{\text{steady state}} = \frac{I}{e} - \cancel{A(N-N_0)P} - \frac{N}{\tau_e}$

$$\Rightarrow \quad \frac{I}{e} = \frac{N}{\tau_e} \quad \Rightarrow \quad N = \frac{\tau_e I}{e}$$

$$\boxed{\text{Below threshold: } N = \frac{\tau_e I}{e}}$$

carrier number increases linearly with increasing current.

Question 2

2.1 If $\bar{N}$, $\bar{P}$ and $\bar{I}$ represent steady state quantities, carry out a small signal analysis for the rate equation from Q.1 for the following perturbations $\delta N$, $\delta P$ and $\delta I$ around these steady state points.

Solution must be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I \\ 0 \end{pmatrix}$$

lec 6. 21 Without modulation:

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N - N_0)P - \frac{P}{\tau_p} + \cancel{B\frac{N}{\tau_e}}$$

Small signal analysis:
Bias current set above threshold
⇒ drop spontaneous emission $\left(B\frac{N}{\tau_e}\right)$

$I = \underline{I} + \delta I \qquad N = \underline{N} + \delta N \qquad P = \underline{P} + \delta P$

$$\frac{d}{dt}(\underline{N}+\delta N) = \frac{\underline{I}+\delta I}{e} - A(\underline{N}+\delta N - N_0)(\underline{P}+\delta P) - \frac{(\underline{N}+\delta N)}{\tau_e}$$

$$\frac{d}{dt}(\underline{P}+\delta P) = A(\underline{N}+\delta N - N_0)(\underline{P}+\delta P) - \frac{(\underline{P}+\delta P)}{\tau_p}$$

Multiplying out

$$\cancel{\frac{d\underline{N}}{dt}} + \frac{d\delta N}{dt} = \cancel{\frac{\underline{I}}{e}} \cancel{- A(\underline{N}-N_0)\underline{P}} \cancel{- \frac{\underline{N}}{\tau_e}} + \frac{\delta I}{e} - A(\underline{N}-N_0)\delta P$$
$$- A\underline{P}\delta N - \cancel{A\delta N\delta P} - \frac{\delta N}{\tau_e}$$

$$\cancel{\frac{d\underline{P}}{dt}} + \frac{d\delta P}{dt} = \cancel{A(\underline{N}-N_0)\underline{P}} \cancel{- \frac{\underline{P}}{\tau_p}} + A(\underline{N}-N_0)\delta P$$
$$+ A\underline{P}\delta N + \cancel{A\delta N\delta P} - \delta P/\tau_p$$

Use: $\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e}$

$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p}$$

and neglect 2nd order terms $(\delta N\delta P = 0)$

$$\Rightarrow \frac{d\delta N}{dt} = \frac{\delta I}{e} - \underline{A(N-N_0)}\,\delta P - A\underline{P}\,\delta N - \frac{\delta N}{\tau_e}$$

$$\frac{d\delta P}{dt} = +\underline{A(N-N_0)}\,\delta P + A\underline{P}\,\delta N - \frac{\delta P}{\tau_p}$$

Above threshold, lasing condition: $A(N-N_0) = 1/\tau_p$

$$\Rightarrow \frac{d\delta N}{dt} = -\frac{\delta P}{\tau_p} - A\underline{P}\,\delta N - \frac{\delta N}{\tau_e} + \frac{\delta I}{e}$$

$$\frac{d\delta P}{dt} = \cancel{\frac{\delta P}{\tau_p}} + A\underline{P}\,\delta N - \cancel{\frac{\delta P}{\tau_p}}$$

$$\Rightarrow \frac{d\delta N}{dt} = -\frac{1}{\tau_p}\,\delta P - \left(A\underline{P} + \frac{1}{\tau_e}\right)\delta N + \frac{\delta I}{e}$$

$$\frac{d\delta P}{dt} = 0\,\delta P + A\underline{P}\,\delta N + 0$$

solution

$$\Rightarrow \boxed{\frac{d}{dt}\begin{pmatrix}\delta N\\ \delta P\end{pmatrix} = \underbrace{\begin{pmatrix}-(A\underline{P}+1/\tau_e) & -1/\tau_p\\ A\underline{P} & 0\end{pmatrix}}_{M}\begin{pmatrix}\delta N\\ \delta P\end{pmatrix} + \begin{pmatrix}\delta I/e\\ 0\end{pmatrix}}$$

$$\Rightarrow \text{If } \frac{d}{dt}\begin{pmatrix}\delta N\\ \delta P\end{pmatrix} = M\begin{pmatrix}\delta N\\ \delta P\end{pmatrix} + \begin{pmatrix}\delta I\\ 0\end{pmatrix}$$

$$\Rightarrow \boxed{M = \begin{pmatrix}-(A\underline{P}+1/\tau_e) & -1/\tau_p\\ A\underline{P} & 0\end{pmatrix}}$$

2.2 Identify all elements of Matrix M

2.3 If $\tilde{\delta N}$, $\tilde{\delta P}$ and $\tilde{\delta I}$ are the expression of the 1st order perturbation of $\delta N$, $\delta P$ and $\delta I$ in the Fourier domain, derive the expressions for $\tilde{\delta N}$ and $\tilde{\delta P}$ as as fn. of $\tilde{\delta I}$.

Fourier Transform: $\delta x(t) \rightarrow \delta x(\Omega)$
$\frac{d}{dt} \rightarrow j\Omega$

$$\frac{d}{dt}\begin{pmatrix} \delta N \\ \delta P \end{pmatrix} = \begin{pmatrix} -(CA\underline{P} + 1/\tau_e) & -1/\tau_p \\ A\underline{P} & 0 \end{pmatrix}\begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

After Fourier Transform...

$$j\Omega\begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(CA\underline{P} + 1/\tau_e) & -1/\tau_p \\ A\underline{P} & 0 \end{pmatrix}\begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} + \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

Want to get expressions for $\tilde{\delta N}$ and $\tilde{\delta P}$

Multiply $j\Omega$ by identity to get 2x2

$$\Rightarrow j\Omega\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} - \begin{pmatrix} -(CAP + 1/\tau_e) & -1/\tau_p \\ AP & 0 \end{pmatrix}\begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix}\left[\begin{pmatrix} j\Omega & 0 \\ 0 & j\Omega \end{pmatrix} + \begin{pmatrix} (CAP + 1/\tau_e) & 1/\tau_p \\ -AP & 0 \end{pmatrix}\right] = \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix}\begin{bmatrix} (j\Omega + AP + 1/\tau_e) & 1/\tau_p \\ -AP & j\Omega \end{bmatrix} = \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

$$\text{Now } M = \begin{pmatrix} j\Omega + AP + 1/\tau_e & 1/\tau_p \\ -AP & j\Omega \end{pmatrix}$$

$$\Rightarrow \quad M \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} \tilde{\delta N} \\ \tilde{\delta P} \end{pmatrix} = M^{-1} \begin{pmatrix} \tilde{\delta I}/e \\ 0 \end{pmatrix}$$

$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Then $M^{-1} = \dfrac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$a = j\Omega + AP + 1/\tau_e \qquad b = 1/\tau_p$$
$$c = -AP \qquad d = j\Omega$$

$$\Rightarrow M^{-1} = \frac{1}{(j\Omega + AP + 1/\tau_e)(j\Omega) - (1/\tau_p)(-AP)} \begin{pmatrix} j\Omega & -1/\tau_p \\ +AP & j\Omega + AP + 1/\tau_e \end{pmatrix}$$

$$\det = j^2\Omega^2 + j\Omega AP + \frac{j\Omega}{\tau_e} + \frac{AP}{\tau_p}$$
$$= -\Omega^2 + \frac{AP}{\tau_p} + j\Omega\left(AP + \frac{1}{\tau_e}\right)$$

$$\Rightarrow M^{-1} = \frac{1}{-\Omega^2 + \frac{AP}{\tau_p} + j\Omega\left(AP + \frac{1}{\tau_e}\right)} \begin{pmatrix} j\Omega & -1/\tau_p \\ AP & j\Omega + AP + \frac{1}{\tau_e} \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$x_1 = a y_1 + b y_2 \qquad x_2 = c y_1 + d y_2$$

$$\Rightarrow \tilde{\delta N} = \frac{j\Omega}{-\Omega^2 + \frac{AP}{\tau_p} + j\Omega\left(AP + \frac{1}{\tau_e}\right)} \left(\frac{\tilde{\delta I}}{e}\right)$$

$$\tilde{\delta P} = \frac{AP}{-\Omega^2 + \frac{AP}{\tau_p} + j\Omega\left(AP + \frac{1}{\tau_e}\right)} \left(\frac{\tilde{\delta I}}{e}\right)$$

Or for $+\Omega^2$ ...

$$\left\{ \begin{aligned} \tilde{\delta N} &= \frac{-j\Omega}{\Omega^2 - \frac{AP}{\tau_p} - j\Omega\left(AP + \frac{1}{\tau_e}\right)} \left(\frac{\tilde{\delta I}}{e}\right) \\ \tilde{\delta P} &= \frac{-AP}{\Omega^2 - \frac{AP}{\tau_p} - j\Omega\left(AP + \frac{1}{\tau_e}\right)} \left(\frac{\tilde{\delta I}}{e}\right) \end{aligned} \right\}$$

From lecture 7:

$$\omega_r^2 = \frac{AP}{\tau_p} \qquad \frac{2}{\tau_r} = AP + \frac{1}{\tau_e}$$

$$\Rightarrow \quad \tilde{\delta N} = \frac{-j\Omega}{\Omega^2 - \omega_r^2 - j\frac{\Omega 2}{\tau_r}} \left(\frac{\tilde{\delta I}}{e}\right)$$

$$\tilde{\delta P} = \frac{-AP}{\Omega^2 - \omega_r^2 - j\frac{\Omega 2}{\tau_r}} \left(\frac{\tilde{\delta I}}{e}\right)$$

$\to \omega_r^2 T_p$

$$\Rightarrow \quad \frac{\tilde{\delta N}}{\tilde{\delta I}} = \frac{-j\Omega}{e\left(\Omega^2 - \omega_r^2 - \frac{2j\Omega}{\tau_r}\right)}$$

$$\frac{\tilde{\delta P}}{\tilde{\delta I}} = \frac{-\omega_r^2 T_p}{e\left(\Omega^2 - \omega_r^2 - \frac{2j\Omega}{\tau_r}\right)}$$

Q3 Consider a photodiode with efficiency $\eta = 0.85$ max at $\lambda = 1550nm$. This PD has BW of 20GHz with load impedance $50\Omega$. temp = 300K and optical power = -3dBm.

Hint: $$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left(2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_L}\right)\Delta f}$$

3.1

For calculations, see overleaf (done in mathematica).

SNR = 44.5dB

3.2 For T = 0°C SNR = 44.77 dB
$\Rightarrow T = 273K$

3.3 If the temp increases, what is the consequence to SNR?

As the temp increases the thermal noise increases. Thus the noise level increases and the SNR decreases.

ClearAll;

In[3]:= h = 6.62 * 10^-34; n = 0.85; PdBm = -3;
T1 = 300; Rc = 50; deltaf = 20 * 10^9; c = 3 * 10^8;
e = 1.6 * 10^-19; k = 1.38 * 10^-23; L = 1550 * 10^-9;

Defining constants

c = f*L
f + c/L
1/f = L/c

getting freq. in terms of wavelength (L) and speed of light (c).

In[4]:= PW = 10^(PdBm / 10) / 1000 → converting dBm to Watts.

Out[4]= $\frac{1}{1000 \times 10^{3/10}}$

In[5]:= N[%, 10]

Out[5]= 0.0005011872336 → -3dBm = 0.0005 watts.

In[10]:=

3.1

$$\frac{\left(\frac{n*e}{h*c/L}\right)^2 * (PW)^2}{\left(2*e*\left(\frac{n*e}{h*c/L}\right)*(PW) + \frac{4*k*T1}{Rc}\right)*(deltaf)}$$

Out[10]= 28 218.9

In[11]:= PdB = 10 * Log10[%] → converting to dB

Out[11]= 44.5054 dB

In[8]:=

$\boxed{\text{For } T = 300k,\ S/R = 44.5 \text{ dB}}$

T2 = 273;

3.2

In[12]:=

$$\frac{\left(\frac{n*e}{h*c/L}\right)^2 * (PW)^2}{\left(2*e*\left(\frac{n*e}{h*c/L}\right)*(PW) + \frac{4*k*T2}{Rc}\right)*(deltaf)}$$

Out[12]= 30 002.5

In[13]:= PdB = 10 * Log10[%]

Out[13]= 44.7716 dB

$\boxed{\text{For } T = 273k,\ S/R = 44.77 \text{ dB}}$

3.4. If the BW increases, what is the consequence for S/R? When the BW increases, this means that the PD can capture more noise. Thus the noise level increases and the S/R decreases.

Qu.1 What is the internal and external quantum efficiency of the LED?

radiative lifetime = 30ns → $\tau_r$
non-radiative lifetime = 45ns → $\tau_{nr}$
refractive index = 3.6 → $n$
emission max. @ $\lambda$ = 450nm → $\lambda$
bias current = 300mA → $I$

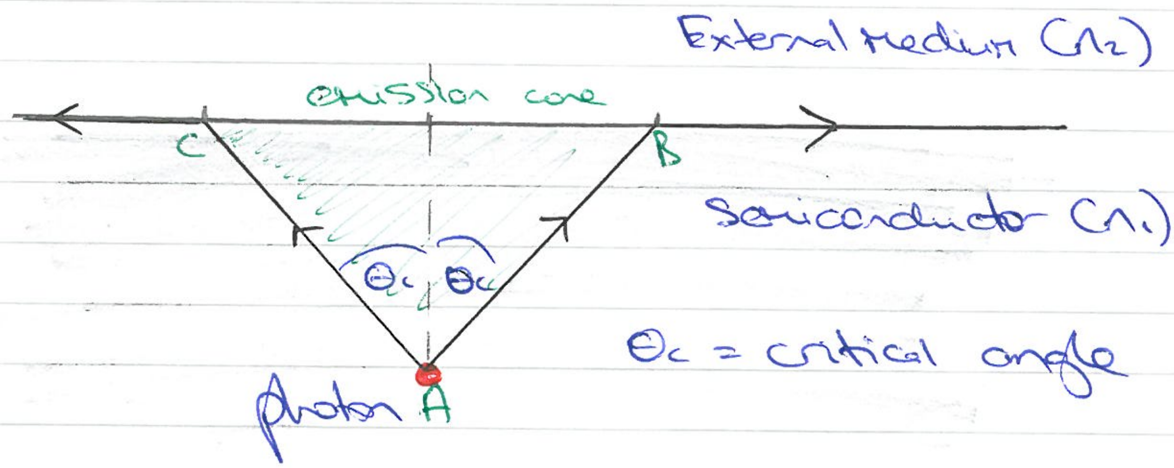
assume: no default in S.C. material and contacts are not light absorbing.

Internal quantum efficiency:

$$\eta_{int} = \frac{1/\tau_r}{1/\tau_r + 1/\tau_{nr}} = \frac{3}{5} = 0.6$$

$$\boxed{\eta_{int} = 0.6}$$

For external efficiency, need to consider solid angle.



Reference: Fiber optics, Prof. R.k. Shevgaonkar, Dept. of Electronic Engineering, Indian I.T. lecture 14 notes, optical Sources.

The quantity $\eta_{ext}$ can be defined as the ratio of the solid angle subtended at A by the emission cone to the total solid angle through which "N" photons have equal probability to travel.

Since the generated photons have equal probabilities to travel along all directions at A, the latter solid angle corresponds to $4\pi$.

$$\Rightarrow \eta_{ext} = \frac{2\pi \int_0^{\theta_c} \sin\theta \, d\theta}{4\pi}$$

$$\eta_{ext} = \frac{1}{2}(1 - \cos\theta_c)$$

Snells law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

At $\theta_1 = \theta_c$, $\theta_2 = 90 \Rightarrow \sin\theta_2 = 1$

$$\Rightarrow \sin\theta_c = \frac{n_2}{n_1} \qquad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1}{3.6}\right) \qquad n_2 = \text{air} = 1, \quad n_1 = 3.6$$

$\Rightarrow \theta_c = 16.127°$

$\Rightarrow \eta_{ext} = 0.019$

$$\boxed{\eta_{ext} \sim 2\%}$$