DUBLIN CITY UNIVERSITY

SEMESTER TWO SOLUTION ASSIGNMENT 2015

MODULE: EE506: Fundamentals of photonic devices

YEAR: Master of Engineering

e	$1.6 \times 10^{-19} C$
t	time
c	$3 \times 10^{8} m/s$
h	$6.62 \times 10^{-34} Js$
k	$1.38 \times 10^{-23} J/K$
cavity length	$250 \mu m$
active region width	$2 \mu m$
active region thickness	$200 nm$
N_o	1.2×10^{8}
A	$8 \times 10^{3} s^{-1}$
τ_p	$1.6 ps$
τ_e	$2.2 ns$
$\frac{dN}{dt}$	time variation of carrier
$\frac{dP}{dt}$	time variation of carrier

1 Question 1 [30 marks]

1.1 [5 marks]

1. **The set of equations given below are called rate equations:**

$$\begin{cases} \frac{dN}{dt} = \frac{I}{e} - A(N - No)P - \frac{N}{\tau_e} \\ \frac{dP}{dt} = A(N - No)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e} \end{cases} \quad (1)$$

What does this set of equations describe? Identify and name all the symbols appearing in Eq. 1. This set of equations describes the time evolution of both the carrier number and the photon number.

Solutions

I is the value of the bias current applied to the laser.

A is the gain coefficient.

N the carrier number.

No the carrier number at transparency.

P the photon number.

τ_e the carrier lifetime.

τ_p the photon lifetime.

β the spontaneous emission coupling.

1.2 [15 marks]

Use Eq. (1) to determine the expression for the current threshold and from the list of constants attached to this exam paper calculate the threshold current. What assumption do you use to facilitate this calculation?

Solutions

To determine the threshold current, the rate equations are used in steady state condition. That means that there is no time variation of the carrier number and photon number, $d/dt = 0$.

At threshold current the spontaneous emission can be neglected. $\beta\frac{N}{\tau_e} \approx 0$.

The gain is equal to the losses, $A(N-No)=\frac{1}{\tau_p}$. The carrier number at threshold current is equal to $(No+\frac{1}{A\tau_p})$.This expression is now used in the carrier number equation.

$\frac{I}{e}-A(N-No)P-\frac{N}{\tau_e}=0$. P can be expressed as $(I-I_{th})$.

The actual expression is $P=\frac{\tau_p}{e}(I-I_{th})$ with $I_{th}=\frac{e}{\tau_e}(No+\frac{1}{A\tau_p})$. P is always positive and is valid only above threshold. Above threshold current there is stimulated emission and the photon number increases with bias current according to a slope $\frac{\tau_p}{e}$.

The value of $I_{th}=14.4mA$.

1.3 [10 marks]

Determine the expression for the number of carriers as a function of the bias current set above threshold and below the threshold. Comment these results.

Solutions

Above threshold the carrier number is clamped to $(No+\frac{1}{A\tau_p})$. The current can increase the carrier number stays at this value, which corresponds to the gain equals to the losses.

Below threshold, the carrier number increases with the current. More and more carriers are injected in the junction.

$N=\frac{\tau_e}{e}I$.

2 Question 2 [30 marks]

2.1 [10 marks]

If $\bar{N}$, $\bar{P}$ and $\bar{I}$ represent steady state quantities, carry out a small signal analysis for the rate equations (1) from Question 1, for the following perturbations δN, δP and δI around these steady state points. Your solution must be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix} \tag{2}$$

where M is a 2×2 matrix.

Solutions

1. We neglect the term of spontaneous emission i.e. $\beta\frac{N}{\tau_e}$

$$\begin{cases} \frac{d(\overline{N}+\delta N)}{dt} = \frac{\overline{I}+\delta I}{e} - A(\overline{N}+\delta N - No)(\overline{P}+\delta P) - \frac{\overline{N}+\delta N}{\tau_e} \\ \frac{d(\overline{P}+\delta P)}{dt} = A(\overline{N}+\delta N - No)(\overline{P}+\delta P) - \frac{(\overline{P}+\delta P)}{\tau_p} \end{cases}$$

This analysis is limited to the first order, so any $\delta N.\ \delta P$ is set to zero.

The equations for the perturbation are written as follows:

$$\begin{cases} \frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(\overline{N} - No)\delta P - (A\overline{P} + \frac{1}{\tau_e})\delta N \\ \frac{d(\delta P)}{dt} = A(\overline{N} - No) - \frac{1}{\tau_p})\delta P + A\overline{P}\delta N \end{cases}$$

If we keep in mind that this study is carried out above threshold, $A(\overline{N} - No) = \frac{1}{\tau_p}$. Then the above equations can be written as:

$$\begin{cases} \frac{d(\delta N)}{dt} = \frac{\delta I}{e} - \frac{\delta P}{\tau_p} - (A\overline{P} + \frac{1}{\tau_e})\delta N \\ \frac{d(\delta P)}{dt} = A\overline{P}\delta N \end{cases}$$

These equations can be written as a matricial equation:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -(A\overline{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\overline{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

2.2 [5 marks]

Identify all the elements of the matrix, M.

Solutions

$$M = \begin{pmatrix} -(A\overline{P} + \frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\overline{P} & 0 \end{pmatrix}$$

2.3 [15 marks]

If $\widetilde{\delta N}$, $\widetilde{\delta P}$ and $\widetilde{\delta I}$ are the expression of the first order perturbation of δN, δP and δI in the Fourier domain, derive the expressions for $\widetilde{\delta N}$ and $\widetilde{\delta P}$ as a function $\widetilde{\delta I}$.

Hint:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{(ad-cb)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{3}$$

Solutions

In the Fourier domain, the elements of the matrix above are invariant, and this equation can be written as:

$$\begin{pmatrix} j\omega\widetilde{\delta N} \\ j\omega\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -(A\overline{P}+\frac{1}{\tau_e}) & -\frac{1}{\tau_p} \\ A\overline{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}$$

This can then be presented as:

$$M\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}.$$

with $M = \begin{pmatrix} j\omega + (A\overline{P}+\frac{1}{\tau_e}) & \frac{1}{\tau_p} \\ -A\overline{P} & j\omega \end{pmatrix}$,

and

$$\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1}\begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix},$$

with

$$M^{-1} = \frac{1}{\omega^2 - \frac{A\overline{P}}{\tau_p} - j\omega(A\overline{P}+\frac{1}{\tau_e})} \begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\overline{P} & -j\omega - (A\overline{P}+\frac{1}{\tau_e}) \end{pmatrix}$$

$\omega_R^2 = \frac{A\overline{P}}{\tau_p}$ and $\frac{2}{\tau_R} = (A\overline{P}+\frac{1}{\tau_e})$.$\omega_R$ is the relaxation oscillation and $\frac{2}{\tau_R}$ the dumping coefficient this oscillation.

$\widetilde{\delta P} = \frac{-A\overline{P}}{\omega^2 - \frac{A\overline{P}}{\tau_p} - j\omega(A\overline{P}+\frac{1}{\tau_e})}\widetilde{\delta I}/e$, thus $\frac{\widetilde{\delta P}}{\widetilde{\delta I}} = \frac{-\tau_p\omega_R^2/e}{\omega^2 - \frac{A\overline{P}}{\tau_p} - j\omega(A\overline{P}+\frac{1}{\tau_e})}$.

$\widetilde{\delta N =} \frac{-j\omega}{\omega^2 - \frac{A\overline{P}}{\tau_p} - j\omega(A\overline{P}+\frac{1}{\tau_e})}\widetilde{\delta I}/e$

3 Question [30 marks]

Consider a photodiode with an efficiency $\eta = 0.85$ maximal at the wavelength of $1550nm$. This photodiode has a bandwidth of $20GHz$ with a load impedance of 50Ω. It works at a temperature of 300^0K and an optical power of $-3dBm$ launched at its input.

Hint:

$$\frac{S}{N} = \frac{(\frac{\eta e}{h\nu})^2 {P_s}^2}{(2e(\frac{\eta e}{h\nu})P_s + \frac{4kT}{R_c})\Delta f}$$

3.1 [10 marks]

Solutions

What is the signal-to-noise ratio of this photodiode?

$\frac{S}{N} = \frac{(\frac{0.85e}{hc/\lambda})^2 {P_s}^2}{(2e(\frac{0.85e}{hc/\lambda})P_s+\frac{4kT}{R_c})\Delta f} = \frac{2.87\times10^{-7}}{(3.4\times10^{-12}+6.62\times10^{-12})} = 44.5dB$

3.2 [10 marks]

What is the value of the signal- to-noise ratio if the temperature is set at 0^oC**?**

Solutions

$\frac{S}{N} = \frac{(\frac{0.85e}{hc/\lambda})^2 {P_s}^2}{(2e(\frac{0.85e}{hc/\lambda})P_s)\Delta f} = \frac{2.87\times10^{-7}}{(3.4\times10^{-12}+6.02\times10^{-12})} = 44.83dB$ S/N is ratio between 2 powers. At the numerator signal power, and at the denominator noise power. This ratio has no unit, so it should be expressed in dB.

3.3 [5 marks]

There is a small increase of S/N with the reduction of temperature. **If the temperature of a photodiode increases, what is the consequence on the signal- to-noise ratio? Justify your answer.**

Solutions

If the temperature increases the thermal noise increases and therefore S/N decreases.

3.4 [5 marks]

If the bandwidth of a photodiode increases, what is the consequence on the signal-to-noise ratio? Justify your answer.

Solutions

If the bandwidth of the photodiode increases, more noise is capture by the photodiode. The level of noise increases and S/N decreases.

[End of Question 3]

4 Question [10 marks]

4.1 [10 marks]

What is the value of the internal and external quantum efficiency of the light emitting diode. The radiative and non-radiative carrier lifetimes are 50ns and 75ns. We assume that there is no default in the semiconductor material and the contacts are not light absorbing. The refractive index of the LED is 3.6 and its emission is maximal at a wavelength of 450nm and bias current of 300 mA.

Solutions

The internal quantum efficiency is given by the ratio of the radiative rate by the sum of the radiative and non-radiative rates:

$$\eta_{int} = \frac{1/50}{\frac{1}{50} + \frac{1}{75}}$$

$$\eta_{int} = 0.6$$

The extraction efficiency is given by the ratio of the spontaneous emission exiting the LED and the total spontaneous emission produced. The efficiency also has to take into account the transmittance at the interface and the internal quantum efficiency.

$$\eta_{extr} = \eta_{int}(1 - (\frac{1-n}{1+n})^2)\frac{2\pi\cos(1-\theta_c)}{4\pi}$$

$$\theta_c = \arcsin{(\frac{1}{n})}$$

$$\theta = 16^o$$

$$\eta_{extr} = 0.3(1 - (\frac{1-n}{1+n})^2)(1 - \cos(\theta_c))$$

$$\eta_{extr} = 0.8\%$$

In this question, we assume that the transmittance did not vary with angle, which can be justified since θ_c is small.

[End of Question 4]