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Declaration

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Programme: MENC (Masters Electronic Systems)

Module Code: EE506

Assignment Title: EE506 Assignment

Submission Date: 24-April-2015

I understand that the University regards breaches of academic integrity and plagiarism as grave and serious.

I have read and understood the DCU Academic Integrity and Plagiarism Policy . I accept the penalties that may be imposed should I engage in practice or practices that breach this policy.

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I declare that this material, which I now submit for assessment, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Name: Kaustav Banerjee

Date: 24-April-2015

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1 Question 1

1.1.

1. The set of equations given below are called rate equations :-

$$\begin{cases} \dfrac{dN}{dt} = \dfrac{I}{e} - A(N-N_0)P - \dfrac{N}{\tau_e} & (1) \\ \dfrac{dP}{dt} = A(N-N_0)P - \dfrac{P}{\tau_p} + \beta\dfrac{N}{\tau_e} \end{cases}$$

What does this set of equation describe? Identify and name all the symbols appearing in Eq. 1 This set of equations describes the time evolution of both the carrier number and the photon number

Solution:-

The set of equations given are :-

$$\frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e}$$

$$\frac{dP}{dt} = A(N-N_0)P - \frac{P}{\tau_p} + \beta\frac{N}{\tau_e}$$

The set of equations describe the following :-

The set of equations applied to the laser.

I = the bias current applied to the laser.

A = Gain coefficient.

N = Number of carriers.

N_0 = Carrier number at transparency.

P = Photon number

τ_e = Carrier lifetime.

τ_p = Lifetime of a photon.

β = Spontaneous emission coupling

$\frac{dN}{dt}$ = Rate of change of carrier numbers with respect to time.

$\frac{dP}{dt}$ = Change in number of photons with respect to time

1.2 Use Eq. (1) to determine the expression for the current threshold and from the list of constants attached to this exam paper, calculate the threshold current. What assumptions do you use to facilitate this calculation?

Solution :-

Let the threshold current be denoted by I_{th}.

The current is given by I.

At steady state condition, the carrier number has no time variation and hence the rate of change of the number of photons is given by

$d/dt = 0$.

At threshold current, there will not be any spontaneous emission. Hence the gain will be equal to the loss incurred.

$$\therefore \quad A(N - N_0) = \frac{1}{\tau_p} \quad — (1).$$

And the carrier number at threshold current is given by :-

$$\left(N_0 + \frac{1}{A\tau_p}\right)$$

$\therefore$ the carrier number equation can be written as :-

$$\frac{I}{e} - A(N - N_0)P - \frac{N}{\tau_e} = 0 \quad — (2).$$

From equation (1), we can write,

$$\frac{I}{e} - P \cdot \frac{1}{\tau_p} - \frac{N}{\tau_e} = 0.$$

$$\text{or,} \quad \frac{P}{\tau_p} = \frac{I}{e} - \frac{N}{\tau_e}$$

$$\therefore P = \tau_p\left(\frac{I}{e} - \frac{N}{\tau_e}\right)$$

Now, the expression for threshold current is given by

$$I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right).$$

From the given set of values, if we substitute for each expression, we get,

$$I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

$$= \frac{1.6\times10^{-19}}{2.2\times10^{-9}}\left(1.2\times10^{8} + \frac{1}{8\times10^{3}\times1.6\times10^{-12}}\right)$$

$$= \frac{1.6\times10^{-19}}{2.2\times10^{-9}}\left(1.2\times10^{8} + \frac{1}{8\times10^{3}\times1.6\times10^{-12}}\right)$$

$$= \frac{1.6\times10^{-19}}{2.2\times10^{-9}}\left(1.2\times10^{8} + \frac{1}{8\times10^{-9}\times1.6}\right)$$

$$= \frac{1.6\times10^{-10}}{2.2}\left(1.2\times10^{8} + \frac{1}{8\times10^{-9}\times1.6}\right)$$

$$= \left(0.73\times10^{-10}\right)\left(1.2\times10^{8} + \frac{1}{12.8\times10^{-9}}\right)$$

$$= \left(0.73\times10^{-10}\right)\left(\frac{1.2\times10^{8}\times12.8\times10^{-9}+1}{12.8\times10^{-9}}\right)$$

$$= \left(0.73\times10^{-10}\right)\left(\frac{\{1.2\times12.8\times10^{-1}\}+1}{12.8\times10^{-9}}\right)$$

$$= \left(0.73\times10^{-10}\right)\left(\frac{\{15.36\times10^{-1}\}+1}{12.8\times10^{-9}}\right)$$

$$= \left(0.73\times10^{-10}\right)\left(\frac{1.536+1}{12.8\times10^{-9}}\right)$$

$$= \frac{0.73\times10^{-10}\times2.536}{12.8\times10^{-9}} = \frac{0.73\times10^{-1}\times2.536}{12.8}$$

$$= \frac{0.073\times2.536}{12.8} = 14.46\ mA \ (\text{Ans}).$$

1.3 Determine the expression for the number of carriers as a function of the bias current set above threshold and below the threshold. Comment these results.

Solution :-

Above threshold value, the carrier number becomes $\left(N_0 + \frac{1}{A\tau_p}\right)$, where N_0 is the carrier number at transparency, A is the gain coefficient and τ_p is the photon lifetime.

At this state, the current can increase but the carrier number stays constant. at ~~this~~

$\therefore$ Gain = loss incurred.

Below threshold value, the carrier number increases with increasing current. More and more carriers are injected in the junction according to the following expression of current:

$$N = \frac{\tau_e}{e} I.$$

where N is the carrier number

τ_e is the carrier lifetime.

I is the bias current applied to the laser &

e is the electron number. (du).

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2 Question 2

2.1 If $\bar{N}$, $\bar{P}$ and $\bar{I}$ represent steady state quantities, carry out a small signal analysis for the rate equations (1) from Question 1, for the following perturbations δN, δP and δI around these steady state points. Your solution must be presented as:

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = M \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \frac{\delta I}{e} \\ 0 \end{pmatrix}$$

where M is a 2X2 matrix.

Solution :-

Here, in this case, the term for spontaneous emission, $\beta \frac{N}{\tau_e}$ can be neglected.

Hence the required expression will be :

$$\frac{d(\bar{N}+\delta N)}{dt} = \frac{\bar{I}+\delta I}{e} - A(\bar{N}+\delta N - N_0)(\bar{P}+\delta P) - \frac{\bar{N}+\delta N}{\tau_e}$$

$\left[\text{from equation (1), } \frac{dN}{dt} = \frac{I}{e} - A(N-N_0)P - \frac{N}{\tau_e} \right]$

And,

$$\frac{d(\bar{P}+\delta P)}{dt} = A(\bar{N}+\delta N - N_0)(\bar{P}+\delta P) - \frac{(\bar{P}+\delta P)}{\tau_e}$$

We need to develop these equations and neglect any second order expression (that is $\delta N \delta P$).

$\therefore$ since these equations are limited to the first order, so any $\delta N \delta P$ should be equal to zero.

The equations for the perturbation are written as follows:-

$$\frac{d(\delta N)}{dt} = \frac{\delta I}{e} - A(\bar{N}-N_0)\delta P - \left(A\bar{P} + \frac{1}{\tau_e}\right)\delta N$$

$$\frac{d(\delta P)}{dt} = A\left\{(\bar{N}-N_0) - \frac{1}{\tau_p}\right\}\delta P + A\bar{P}\delta N$$

In this case, if the current is considered to be above threshold value, $A(\bar{N} - N_0) = \frac{1}{\tau_p}$, then the equations can be written as :-

$$\left.\begin{aligned} \frac{d(\delta N)}{dt} &= \frac{\delta I}{e} - \frac{\delta P}{\tau_p} - \left(A\bar{P} + \frac{1}{\tau_e}\right)\delta N \\ \frac{d(\delta P)}{dt} &= A\bar{P}\,\delta N \end{aligned}\right\}$$

Now, if we write these equations in matrix format, the given representation would be :-

$$\begin{pmatrix} \frac{d\delta N}{dt} \\ \frac{d\delta P}{dt} \end{pmatrix} = \begin{pmatrix} -\left(A\bar{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \delta N \\ \delta P \end{pmatrix} + \begin{pmatrix} \delta I/e \\ 0 \end{pmatrix}$$

$\therefore$ from the above matrix representation, the value of M can be derived as :-

$$M = \begin{pmatrix} -\left(-A\bar{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \quad (\text{Ans}).$$

2.2 Identify all the elements of the matrix, M.

Solution:-

$$M = \begin{pmatrix} -\left(A\underline{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\underline{P} & 0 \end{pmatrix} \quad (\text{Ans}).$$

2.3 If $\widetilde{\delta N}$, $\widetilde{\delta P}$ and $\widetilde{\delta I}$ are the expression of the first order perturbation of δN, δP and δI in the Fourier domain, derive the expressions for $\widetilde{\delta N}$ and $\widetilde{\delta P}$ as a function $\widetilde{\delta I}$

hint:
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{(ad-bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Solution :

In the Fourier domain, the elements of the matrix given in the above expression should be invariant. So in this case, the equation can be written as :-

$$\begin{pmatrix} j\omega\widetilde{\delta N} \\ j\omega\widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} -\left(A\bar{P} + \frac{1}{\tau_e}\right) & -\frac{1}{\tau_p} \\ A\bar{P} & 0 \end{pmatrix} \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} + \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix} \quad \text{—} (1)$$

This can be represented as:

$$M \begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}.$$

Now, for this case,

$$M = \begin{pmatrix} j\omega + \left(A\bar{P} + \frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\bar{P} & j\omega \end{pmatrix}$$

and
$$\begin{pmatrix} \widetilde{\delta N} \\ \widetilde{\delta P} \end{pmatrix} = M^{-1} \begin{pmatrix} \widetilde{\delta I}/e \\ 0 \end{pmatrix}$$

with

$$M^{-1} = \frac{1}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega\left(A\bar{P} + \frac{1}{\tau_e}\right)} \begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\bar{P} & -j\omega - \left(A\bar{P} + \frac{1}{\tau_e}\right) \end{pmatrix} \quad \omega_R^2 = \frac{A\bar{P}}{\tau_p}$$

And, $\frac{2}{\tau_R} = \left(A\bar{P} + \frac{1}{\tau_e}\right)$. ω_R is the relaxation oscillation

$\frac{2}{\tau_R}$ is the dumping coefficient of this oscillation.

$$\therefore\ \delta\tilde{P} = \frac{-AP}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega\left(A\bar{P} + \frac{1}{\tau_e}\right)}\ \delta\tilde{I}/e$$

$$\text{So, } \frac{\delta\tilde{P}}{\delta\tilde{I}} = \frac{-\tau_p \omega_R^2/e}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega\left(A\bar{P} + \frac{1}{\tau_e}\right)}$$

$$\delta\tilde{N} = \frac{-j\omega}{\omega^2 - \frac{A\bar{P}}{\tau_p} - j\omega\left(A\bar{P} + \frac{1}{\tau_e}\right)} \cdot \delta\tilde{I}/e.$$

Again, equation (1) is in the form, $MV = B$, where

$$M = \begin{pmatrix} j\omega + \left(A\bar{P} + \frac{1}{\tau_e}\right) & \frac{1}{\tau_p} \\ -A\bar{P} & j\omega \end{pmatrix}$$

$$V = \begin{pmatrix} \delta\tilde{N} \\ \delta\tilde{P} \end{pmatrix}$$

$$B = \begin{pmatrix} \delta\tilde{I}/e \\ 0 \end{pmatrix}.$$

Multiplying both sides of equation (1), by inverse matrix of M,

$M^{-1}MV = M^{-1}B$

$\therefore\ V = M^{-1}B.$

Now this is in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{(ad-bc)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

So from this, the value of M^{-1} comes to be:

$$M^{-1} = \frac{1}{\left(\left(j\omega + \left(A\underline{P} + \frac{1}{\tau_s}\right)\right)(j\omega)\right) - \left(\left(\frac{1}{\tau_p}\right)\left(-A\underline{P}\right)\right)} \begin{pmatrix} j\omega & -\frac{1}{\tau_p} \\ -A\underline{P} & j\omega + \left(A\underline{P} + \frac{1}{\tau_s}\right) \end{pmatrix}$$

3 Question

Consider a photodiode with an efficiency $\eta = 0.85$ maximal at the wavelength of 1550 nm. This photodiode has a bandwidth of 20 GHz with a load impedance of 50 Ω. It works at a temperature of 300 K and an optical power of −3 dBm launched at its input.

Hint :

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left\{2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_L}\right\}\Delta f}$$

3.1 What is the signal-to-noise ratio of this photodiode?

Solution :

Given,

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{h\nu}\right)^2 P_s^2}{\left\{2e\left(\frac{\eta e}{h\nu}\right)P_s + \frac{4kT}{R_L}\right\}\Delta f}$$

Now, as we know from formula that $\nu = c/\lambda$, hence substituting the values, we get,

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{hc/\lambda}\right)^2 P_s^2}{\left\{2e\left(\frac{\eta e}{hc/\lambda}\right).P_s + \frac{4kT}{R_L}\right\}\Delta f}$$

Now, it is given that,

$\eta = 0.85$

$e = 1.6 \times 10^{-19}$ C

$h = 6.62 \times 10^{-34}$ J/S

$P_s = -3$ dB

$k = 1.38 \times 10^{-23}$ J/K

Temperature $T = 300$ K

Bandwidth $\Delta f = 20$ GHz.

Wavelength $\lambda = 1550$ nm $= 1550 \times 10^{-9}$ m.

And we already know that $c = 3 \times 10^{8}$ m/s

Now substituting these expressions in the formula for $\frac{S}{N}$, we get,

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{hc/\lambda}\right)^2 . P_s^2}{\left\{2e\left(\frac{\eta e}{hc/\lambda}\right). P_s + \frac{4kT}{R_L}\right\}\Delta f}$$

$$= \frac{\left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-19} \times 3 \times 10^8 / 1550 \times 10^{-9}}\right).(-3)^2}{\left\{2 \times \cancel{3 \times 10^{-8}}\,(1.6 \times 10^{-19})\left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-19} \times 3 \times 10^8 / 1550 \times 10^{-9}}\right)(-3) + \frac{4 \times 1.38 \times 10^{-23} \times 300}{50}\right\}. \times (20 \times 10^9)^{\times 20}}$$

$$= \frac{\left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^8 / 1550 \times 10^{-9}}\right).(.000501187)^2}{\left\{2 \times (1.6 \times 10^{-19})\left(\frac{0.85 \times 1.6 \times 10^{-19}}{\frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1550 \times 10^{-9}}}\right)(.000501187) + \frac{4(1.38 \times 10^{-23})(300)}{50}\right\}(20 \times 10^9)}$$

3.

On calculation, we get,

$\frac{S}{N} = 44.505$ dB. (Ans).

[formula for conversion of dB to mW used: $dBm = \log_{10}(mW) * 10$
& $mW = \cancel{1000\,mW}\ 10\hat{}(dBm/10)$]

3..

3.2 What is the value of the signal-to-noise ratio if the temperature is set to 0°C?

Solution:

When the temperature is set to 0°C, converting it to Kelvin, we get 0°C = 273 K.

∴ the required signal-to-noise ratio would be:

$$\frac{S}{N} = \frac{\left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-19} \times 3 \times 10^{8} / 1550 \times 10^{-9}}\right) \cdot \left(.000501187\right)^{2}}{\left\{ 2 \times (1.6 \times 10^{-19}) \left(\frac{0.85 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-19} \times 3 \times 10^{8} / 1550 \times 10^{-9}}\right)\left(.000501187\right) + \frac{4(1.38 \times 10^{-23})(273)}{50} \right\}\left(20 \times 10^{9}\right)}$$

On calculation, we get,

$\frac{S}{N} = 44.77$ dB (~~ute~~).

[formula for conversion of dB to mW used: dBm = $\log_{10}(mW) * 10$
$mW = 1\cancel{000} \cdot 10$^$(dBm/10)$]

3.3 If the temperature of a photodiode increases, what is the consequence on the signal-to-noise ratio? Justify your answer.

Solution:

When the temperature of the photodiode increases, the signal-to-noise ratio decreases.

3.4 If the bandwidth of a photodiode increases, what is the consequence on the signal-to-noise ratio? Justify your answer.

Solution:

When the bandwidth of a photodiode increases, the signal-to-noise ratio decreases.

4 Question

4.1 What is the value of the internal and external quantum efficiency of the light emitting diode. The radiative and non-radiative carrier lifetimes are 50ns and 75ns. We assume that there is no default in the semiconductor material and the contacts are not light absorbing. The refractive index of the LED is 3.6 and its emission is maximal at a wavelength of 450nm and bias current of 300mA.

Solution :-

Radiative lifetime = 50ns = 50×10^{-9} sec

Non-radiative lifetime = 75ns = 75×10^{-9} sec

Refractive index of the LED = 3.6

Wavelength = 450nm

Bias current = 300mA

Now the internal quantum efficiency is given by:-

$$\eta_{int} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}}$$

$$= \frac{75 \times 10^{-9}}{50 \times 10^{-9} + 75 \times 10^{-9}} = 0.6.$$

Now the Optical power $P = \eta_{ext}\, \eta_{int} \frac{h\nu I}{e}$

$$= \eta_{ext}\, \eta_{int} \frac{\frac{hc}{\lambda} . I}{e}$$

$$= \eta_{ext} . \eta_{int} . \frac{hcI}{\lambda e}.$$

∴ the extraction efficiency is given by :

$$\eta = \eta_{int} . K . \left[\frac{1}{(n+1)^2}\right] . \frac{1}{n} = 0.6 . 1 \left[\frac{1}{(3.6+1)^2}\right] . \frac{1}{3.6}$$

$$= 0.6 \left[\frac{1}{(4.6)^2}\right] . \frac{1}{3.6}$$

$$= 0.67 \;(\text{ans}).$$

[Here $n = 3.6$ and $K = 1$ since the contacts are not light absorbing]