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Declaration

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Programme: M.Eng in Electronic Systems

Module Code: EE506

Assignment Title: EE506 Assignment 2015

Submission Date: 24-04-15

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EE506 Fundamentals of Photonic Devices assignment

Question 1

1.1 Rate equations

Equation 1 is 1st order partial differential equation describing the time evolution of the carrier number.

$$\frac{dN}{dt} = \frac{I}{e} - A(N - N_O)P - \frac{N}{\tau_e}$$

Equation 1

Where:

N = No. of carriers

I = No. of charge carriers per unit time

e = Elementary charge

$\frac{I}{e}$ = Carriers injected by the bias current per unit time

A = Slope of the linear dependence

N_O = Carrier at transparency

$A(N-N_O)$ = Gain expressed as a linear function of the carrier, also written as G

$A(N-N_O)P$ = Carriers consumed by the radiative recombination due to the stimulated emission

P = No. of photons – required to induce a transition of an electron from the conduction band to the valence band. There are GP carries relaxing from conduction band to valence band.

$\frac{N}{\tau_e}$ = Spontaneous emission – carries recombining spontaneously or lost in defaults in the crystal.

τ_e = amount of time carriers stay in the conduction band.

Equation 2 is 1st order partial differential equation describing the time evolution of the photon number.

$$\frac{dP}{dt} = A(N - N_O)P - \frac{P}{\tau_P} + \beta\frac{N}{\tau_e}$$

Equation 2

Where:

P = No. of photons

$\frac{P}{\tau_p}$ = rate of photons lost by the transmission of the facets and by imperfections of the waveguide

β = photons coupled to the mode

1.2 Expression for current threshold

The rate equations are in steady state condition, which means $\frac{dN}{dt}$ and $\frac{dP}{dt}$ are set to 0.

$$\frac{I}{e} - A(N - N_O)P - \frac{N}{\tau_e} = 0$$

Equation 3

$$A(N - N_O)P - \frac{P}{\tau_P} + \beta\frac{N}{\tau_e} = 0$$

Equation 4

At threshold current, the spontaneous emission ($\beta\frac{N}{\tau_e}$) can be neglected, therefore eqn 4 becomes:

$$A(N - N_O)P - \frac{P}{\tau_P} = 0$$

Equation 5

$$A(N - N_O)P = \frac{P}{\tau_P}$$

Divide both sides by P, to get gain:

$$A(N - N_O) = \frac{1}{\tau_P} = G$$

Equation 6

Sub in eqn 6 into eqn 3:

$$\frac{I}{e} - \frac{1}{\tau_P}P - \frac{N}{\tau_e} = 0$$

Equation 7

P is expressed as:

$$P = \frac{\tau_P}{e}(I - I_{th})$$

Equation 8

Sub in eqn 8 into eqn 7:

$$\frac{I}{e} - \frac{1}{\tau_P}\left(\frac{\tau_P}{e}(I - I_{th})\right) - \frac{N}{\tau_e} = 0$$

$$\frac{I}{e} - \frac{I + I_{th}}{e} - \frac{N}{\tau_e} = 0$$

$$\frac{I_{th}}{e} - \frac{N}{\tau_e} = 0$$

$$\frac{I_{th}}{e} = \frac{N}{\tau_e}$$

$$I_{th} = \frac{e}{\tau_e} N$$

Equation 9

The carrier number at threshold current is:

$$N = N_0 + \frac{1}{A\tau_p}$$

Equation 10

Sub in eqn 10 into eqn 9:

$$I_{th} = \frac{e}{\tau_e}\left(N_0 + \frac{1}{A\tau_p}\right)$$

Equation 11

Eqn 11 is the expression for current threshold.

The current threshold is calculated for the following parameters:

e = 1.6×10^{-19}C

N_O = 1.2×10^{8}

A = 8×10^{3} s^{-1}

τ_P = 1.6 ps

τ_e = 2.2 ns

$$I_{th} = \frac{1.6x10^{-19}}{2.2x10^{-9}}\left(1.2x10^{8} + \frac{1}{(8x10^{3})(1.6x10^{-12})}\right) = 14.409\ mA$$

1.3 Expression for no. of carriers

Above threshold, the carrier number is clamped:

$$N = N_O + \frac{1}{A\tau_P}$$

Even if the current increases, N stays the same as it corresponds to the gain equal to the losses.

Blow threshold, N increases as the current increases. More carriers are injected in the junction according to this linear expression with current.

$$N = \frac{\tau_e}{e} I$$

Question 2

2.1 Small signal analysis on rate equations

In small signal analysis, we introduce a perturbation of the current = δI

This results in perturbation of the carrier number and the photon number = δN and δP respectively.

In the rate equations (eqn 1 and eqn 2), substitute I+δI for I, N+δN for N and P+δP for P. Therefore the rate equations are now:

$$\frac{d\underline{\mathrm{N}} + \delta\mathrm{N}}{dt} = \frac{\underline{\mathrm{I}} + \delta\mathrm{I}}{e} - A(\underline{\mathrm{N}} + \delta\mathrm{N} - N_O)\,(\underline{\mathrm{P}} + \delta\mathrm{P}) - \frac{\underline{\mathrm{N}} + \delta\mathrm{N}}{\tau_e}$$

Equation 12

$$\frac{d\underline{\mathrm{P}} + \delta\mathrm{P}}{dt} = A(\underline{\mathrm{N}} + \delta\mathrm{N} - N_O)\,(\underline{\mathrm{P}} + \delta\mathrm{P}) - \frac{\underline{\mathrm{P}} + \delta\mathrm{P}}{\tau_P} + \beta\frac{\underline{\mathrm{N}} + \delta\mathrm{N}}{\tau_e}$$

Equation 13

The small signal analysis is only performed on 1st order expressions, 2nd order expressions are neglected and the spontaneous emission is also neglected.

Eqn 12 becomes:

$$\frac{d\delta\mathrm{N}}{dt} = \frac{\delta\mathrm{I}}{e} - \left(A\underline{\mathrm{P}} + \frac{1}{\tau_e}\right)\delta\mathrm{N} - \frac{\delta\mathrm{P}}{\tau_P}$$

Equation 14

Eqn 13 becomes:

$$\frac{d\delta\mathrm{P}}{dt} = \left(A(\underline{\mathrm{N}} - N_O) - \frac{1}{\tau_p}\right)\delta\mathrm{P} + A\underline{\mathrm{P}}\delta\mathrm{N}$$

Equation 15

The analysis is above threshold, therefore: $A(N-N_O)=\frac{1}{\tau_P}$, then eqn 15 becomes:

$$\frac{d\delta\mathrm{P}}{dt}=A\underline{\mathrm{P}}\delta\mathrm{N}$$

Equation 16

Eqn 14 and 16 represented in a matrix:

$$\begin{pmatrix}\frac{d\delta\mathrm{N}}{dt}\\ \frac{d\delta\mathrm{P}}{dt}\end{pmatrix}=\begin{pmatrix}-\left(A\underline{\mathrm{P}}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p}\\ A\underline{\mathrm{P}} & 0\end{pmatrix}\begin{pmatrix}\delta\mathrm{N}\\ \delta\mathrm{P}\end{pmatrix}+\begin{pmatrix}\frac{\delta\mathrm{I}}{e}\\ 0\end{pmatrix}$$

Equation 17

2.2 Identify all elements of matrix, M.

$$\mathrm{M}=\begin{pmatrix}-\left(A\underline{\mathrm{P}}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p}\\ A\underline{\mathrm{P}} & 0\end{pmatrix}$$

2.3 Fourier domain expressions for $\widetilde{\delta\mathrm{N}}$ and $\widetilde{\delta\mathrm{P}}$ as a function of $\widetilde{\delta\mathrm{I}}$

Converting eqn17 to Fourier domain gives:

$$\begin{pmatrix}j\omega\widetilde{\delta\mathrm{N}}\\ j\omega\widetilde{\delta\mathrm{P}}\end{pmatrix}=\begin{pmatrix}-\left(A\underline{\mathrm{P}}+\frac{1}{\tau_e}\right) & -\frac{1}{\tau_p}\\ A\underline{\mathrm{P}} & 0\end{pmatrix}\begin{pmatrix}\widetilde{\delta\mathrm{N}}\\ \widetilde{\delta\mathrm{P}}\end{pmatrix}+\begin{pmatrix}\frac{\widetilde{\delta\mathrm{I}}}{e}\\ 0\end{pmatrix}$$

$$\begin{pmatrix}j\omega+\left(A\underline{\mathrm{P}}+\frac{1}{\tau_e}\right) & \frac{1}{\tau_p}\\ -A\underline{\mathrm{P}} & j\omega\end{pmatrix}\begin{pmatrix}\widetilde{\delta\mathrm{N}}\\ \widetilde{\delta\mathrm{P}}\end{pmatrix}=\begin{pmatrix}\frac{\widetilde{\delta\mathrm{I}}}{e}\\ 0\end{pmatrix}$$

Equation 18

Eqn 18 is in form MV = B, where:

$$\mathrm{M}=\begin{pmatrix}j\omega+\left(A\underline{\mathrm{P}}+\frac{1}{\tau_e}\right) & \frac{1}{\tau_p}\\ -A\underline{\mathrm{P}} & j\omega\end{pmatrix}$$

$$\mathrm{V}=\begin{pmatrix}\widetilde{\delta\mathrm{N}}\\ \widetilde{\delta\mathrm{P}}\end{pmatrix}$$

$$\mathrm{B}=\begin{pmatrix}\frac{\widetilde{\delta\mathrm{I}}}{e}\\ 0\end{pmatrix}$$

Multiply both sides of eqn 18 by inverse of matrix M:

$M^{-1}MV = M^{-1}B$

⇨ $V = M^{-1}B$

Find M^{-1}:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{(ad-bc)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$M^{-1} = \frac{1}{\left(\left(j\omega + \left(A\underline{P} + \frac{1}{\tau_e}\right)\right)(j\omega)\right) - \left(\left(\frac{1}{\tau_p}\right)\left(-A\underline{P}\right)\right)}\begin{pmatrix} j\omega & -\frac{1}{\tau_p} \\ A\underline{P} & j\omega + \left(A\underline{P} + \frac{1}{\tau_e}\right) \end{pmatrix}$$

$$M^{-1} = \frac{1}{-\omega^2 + \left(A\underline{P} + \frac{1}{\tau_e}\right)j\omega + \frac{A\underline{P}}{\tau_p}}\begin{pmatrix} j\omega & -\frac{1}{\tau_p} \\ A\underline{P} & j\omega + \left(A\underline{P} + \frac{1}{\tau_e}\right) \end{pmatrix}$$

$$M^{-1} = \frac{1}{\omega^2 - \left(A\underline{P} + \frac{1}{\tau_e}\right)j\omega - \frac{A\underline{P}}{\tau_p}}\begin{pmatrix} -j\omega & \frac{1}{\tau_p} \\ -A\underline{P} & -j\omega - \left(A\underline{P} + \frac{1}{\tau_e}\right) \end{pmatrix}$$

$\omega_R^2 = \frac{A\underline{P}}{\tau_p}$ *and* $\frac{2}{\tau_R} = A\underline{P} + \frac{1}{\tau_e}$. ω_R is the relaxation oscillation and $\frac{2}{\tau_R}$ the dumping coefficient this oscillation.

$$\widetilde{\delta P} = \frac{-A\underline{P}}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)}\left(\frac{\widetilde{dI}}{e}\right)$$

$$\frac{\widetilde{\delta P}}{\widetilde{\delta I}} = \frac{\frac{-\tau_p\omega_R^2}{e}}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)}$$

$$\widetilde{\delta N} = \frac{-j\omega}{\omega^2 - \frac{A\underline{P}}{\tau_p} - j\omega\left(A\underline{P} + \frac{1}{\tau_e}\right)}\left(\frac{\widetilde{dI}}{e}\right)$$

Question 3

3.1 Signal-to-Noise Ratio of photodiode

The parameters for the photodiode are:

Efficiency η = 0.85

Wavelength = 1550nm

Bandwidth = $20x10^9$ Hz

Load impedance = 50Ω

Temperature = 300 K

Power = -3dBm

$$\frac{S}{N} = \frac{\left(\frac{\eta e}{hv}\right)^2 P_S^2}{\left(2e\left(\frac{\eta e}{hv}\right)P_S + \frac{4kT}{R_C}\right)\Delta f}$$

$$\frac{S}{N} = \frac{\left(\frac{(0.85)(1.6x10^{-19})}{\frac{(6.62x10^{-34})(3x10^8)}{1550x10^{-9}}}\right)^2 (.000501187)^2}{\left(2(1.6x10^{-19})\left(\frac{(0.85)(1.6x10^{-19})}{\frac{(6.62x10^{-34})(3x10^8)}{1550x10^{-9}}}\right)(.000501187) + \frac{4(1.38x10^{-23})(300)}{50}\right)(20x10^9)}$$

$$\frac{S}{N} = 44.505\ dB$$

3.2 Signal-to-Noise ratio when temp = 0°C

Temp = 0^oC=273K

$$\frac{S}{N} = 44.77\ dB$$

3.3 If temperature of a photodiode increases, what is the consequence on the SNR?

When the temperature of a photodiode increases, the thermal noise and noise level increases and the SNR decreases.

3.4 If bandwidth of a photodiode increases, what is the consequence on the SNR?

When the bandwidth of a photodiode increases, the photodiode can capture more noise, therefore the noise level increases and the SNR decreases.

Question 4

4.1 Internal and external quantum efficiency of LED

The parameters for the LED are:

Refractive index of LED = 3.6

Wavelength = 450nm

Bias current = 300mA

Radiative carrier lifetime = 50ns

Non-Radiative carrier lifetime = 75ns

Internal efficiency:

$$\eta_{int} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}}$$

$$\eta_{int} = \frac{75x10^{-9}}{50x10^{-9} + 75x10^{-9}} = 0.6$$

Extraction efficiency: