Problem Set 2: Basic Principles of Cellular Systems

1) Consider a city of 10 square kilometres. The cellular system design divides the city into square cells of 1 square kilometre, where each cell can accommodate 100 users.
    a. Find the total number of users that can be accommodated in the system and the length of time it takes a mobile user to traverse a cell assuming they are moving at 30km/hr.
    b. Repeat a. with the cell size reduced to 100 square metres and everything in the system scaled so that 100 users can be accommodated in each cell.

Solution:

a. There are 10 cells each of which can accommodate 100 users. Therefore, the total capacity of the system is 1000 users. Assuming that a user is going from one edge to the edge opposite, a cell will be traversed in 2 minutes.
b. $10km^2 = 10^7\ m^2$. Thus we need 10^5 cells to cover the area. Assuming 100 users per cell, the system capacity is now 10^7 users.

2) Consider a cellular system with diamond-shaped cells of radius R=100m. Suppose the minimum distance between cell centres using the same frequency is D=600m to maintain required SIR.
    a. Find the required reuse factor N.
    b. If the total number of channels in the system is 450, find the number of channels that can be assigned to each cell.

Solution:

a. In general, cell area is $(2R)^2 = 4R^2$ and cluster area is $(2(1/2D))^2 = D^2$.
   So, $N = 0.25(D/R)^2$. In this case, this gives N = 9.
b. The number of channels per cell is Total/N = 50.

3) Consider a cellular system with hexagonal cells of radius R=1km. Suppose the minimum distance between cell centres using the same frequency is D=6km to maintain the required SIR.
    a. Find the required reuse factor N.
    b. If the total number of channels for the system is 1200, find the number of channels that can be assigned to each cell.

Solution:

a. From notes, for hexagonal cells, $N=1/3(D/R)^2$. Thus, N=12.
b. The number of channels per cell is Total/N = 100.

4) Consider an orthogonal system of diamond shaped cells.
    a. Compute the SIR as a function of cell radius R, reuse distance D, and γ.
    b. Compute the SIR for R=10m, D=60m, and $\gamma=2$
    c. Repeat b. for $\gamma=3$.
    d. Repeat b for D=30.
    e. Repeat b for R=15.

Solution:

a. Assume that only 1st tier cells contribute to interference. There are 8 cells to consider. For simplicity, we will assume distances D and R. Thus SIR $= 1/8(D/R)^\gamma$.
b. $SIR=1/8(60/10)^2 = 4.5$
c. $SIR=1/8(60/10)^3 = 27$
d. $SIR=1/8(30/10)^2 = 11.25$
e. $SIR=1/8(60/15)^2 = 2$

5) Consider an orthogonal system of hexagonal shaped cells.
    a. Compute the SIR for R=10m, D=60m, and $\gamma=2$
    b. Repeat b. for $\gamma=3$.
    c. Repeat b for D=30.
    d. Repeat b for R=15.
    e. Explain the above results in terms of the cochannel interference reducing factor Q.

Solution:

a. $SIR = 1/6 (D/R)^\gamma = 6$.
b. SIR = 36
c. SIR = 4.5
d. SIR = 1.33
e. Q=D/R. So, the higher Q, the higher the SIR. For parts b, c, d Q is 6, 3, and 2 respectively.

6) Consider an orthogonal system with hexagonal cells and $\gamma=3$.
    a. Find the minimum reuse factor needed for an SIR of 20dB.
    b. Find the user capacity assuming a total system bandwidth of 20MHz and required signal bandwidth of 100kHz.

Solution:

a. N is $1/3(6SIR)^{2/\gamma}$. 20dB corresponds to SIR=100. So N=24. The closest we can get to this with whole I and J is 27.
b. 200 channels are possible. Thus, we can have 7 channels per cell.

7) Consider an orthogonal system with diamond shaped cells and γ=3.
   a. Find the minimum reuse factor needed for an SIR of 20dB.
   b. Find the user capacity assuming a total system bandwidth of 20MHz and required signal bandwidth of 100kHz.

Solution:

a. We have $N = 0.25(D/R)^2$. So, $(D/R)=(4N)^{1/2}$. Plugging in we get, $SIR = 1/8(4N)^{3/2}$. 20dB corresponds to SIR=100, so $100 = 1/8(4N)^{3/2}$ or N=21.54 or 22. Since we want square cells, we move to N = 25.
b. 20,000,000/100,000 = 200 channels are available. Thus we can have 8 channels per cell.

8) Using the table at the end of this tutorial, approximate the answers to the following (use straight line interpolation). Also, describe in words the general problem being solved.
   a. Given N=20, A=10.5, find P.
   b. Given N=20, P=0.015, find A.
   c. Given P=0.005, A=6, find N.

Solution:

a. A is between 11.09 (P=0.005) and 9.412 (P=0.001). (0.005-0.001)/(11.09-9.412)=(P-0.001)/(10.5-9.412). Solving we get P=0.0036. The question being asked is given a number of resources N and a load A, what is the probability that load will not be served.
b. P is between 0.02 or 2% (A=12.03) and 0.01 or 1% (A=13.18). Since it is halfway, it is easy to see A=12.605. The question being asked is given a number of resources and a target loss (grade of service), what traffic load can be handled?
c. The A required need between 13 and 14 trunks. Since you can't have fractional trunks, pick 14. The question being asked is given a target grade of service and predicted load, how many resources must be provided.

9) The one-way bandwidth available to a single operator in the AMPS system is 12.5MHz with a channel bandwidth of 30kHz and 21 control channels. Consider an AMPS system with the following parameters:

- Cell area = 8km^2
- Total coverage area = 4000km^2
- Frequency reuse factor = 7
- Average number of calls per user during the busy hour = 1.2
- Average holding time of a call = 100s
- Call blocking probability is 2%

a. How many voice channels are there per cell?
b. Using table at end of tutorial, determine the total traffic carried per cell in Erlangs/cell. Convert this to Erlangs/km.
c. Calculate the number of calls/hour/cell and the numbers of calls/hour/km^2.
d. Calculate the number of users/hour/cell and the number of users/hour/channel.

Solution:

a. There are a total of 416 channels available. With a reuse factor of 7, this gives 59 channels per cell. If 21 of these are control, 38 may be used for voice.
b. Using the table at the end of the document, for each cell we have traffic A=29.17 Erlangs/cell. This gives A=3.65 Erlangs/km^2 .
c. Each cell has a load of 29.17 Erlangs. Recall, that an Erlang is the number of call requests during the duration of one call which here is 100sec. there are 3600 seconds per hour giving 1050.12 requests per hour per cell. On average 98% of these are made giving 1029.12 calls/hour/cell. Similarly, we get 131.27 requests/hour/km^2 or 128.64 calls/hour/km^2
d. If each user makes on average 1.2 calls, we get 1029.12/1.2=857.6 users/hour/cell and 128.79/1.2= 107.2 users/hour/km^2 .

10) Consider a 7-cell system covering an area of $3100km^2$. The traffic in the seven cells is as follows:

Cell number	1	2	3	4	5	6	7
Traffic (Erlangs)	30.8	66.7	48.6	33.2	38.2	37.2	32.6

Each user generates an average of 0.03 Erlangs of traffic per hour, with a mean holding time of 120s. The systems consists of a total of 395 channels and is designed for a grade of service of 0.02.

a. Determine the number of subscribers in each cell.
b. Determine the number of calls per hour per subscriber.
c. Determine the number of calls per hour in each cell.
d. Using the table at the end of the tutorial, determine the number of channels required in each cell.
e. Determine the total number of subscribers.
f. Determine the average number of subscribers/channel.
g. Determine the subscriber density per km^2.
h. Determine the total traffic in Erlangs
i. Determine the Erlangs per km^2.
j. What is the radius of a cell?

Solution:

a. subscribers per cell = traffic per cell/traffic per user.

Cell number	1	2	3	4	5	6	7
Traffic (Erlangs)	30.8	66.7	48.6	33.2	38.2	37.2	32.6
subscribers	1026.7	2223.3	1620	1106.7	1273.3	1240	1086.7

b. Subscribers make 0.03 call requests every 120s. Thus they make 0.9 requests per hour of which on average 98% connect. This gives 0.882 calls per hour per subscriber.
c. Calls per hour per cell is subscribers per cell x calls per hour per subscriber. E.g. for cell 1 we have 1026.7x0.882=905.5 calls per hour per cell.
d. Again looking at cell 1, we have traffic = 30.8 erlangs and we need a grade of service of 0.02 (or 2%). Using the table at the end, we need 40 channels per cell.
e. Total number of subscribers can be obtained by adding up results of part c.
f. The average number of subscribers per channel is given by total number of subscribers/total number of channels, where the total number of channels is 40x7.
g. The subscriber density is the total number of subscribers/3100.
h. Total traffic is the sum of traffic per cell.
i. $Erlangs/km^2$ is total traffic/3100
j. Assuming equal size cells, the size of a cell is $3100/7=443km^2$. Further assuming that the cells are hexagonal, we get $area=3squrt(3)R^2/2=443$. Solving for R we get R=13.1km.

11) A certain mobile network requires an SIR of 18dB. Measures in a urban area show that the power of the received signal drops with d^4. The system requires GoS=0.01 and has 126 distinct channels.

What is the minimum possible number of hexagon cells in a cluster for this system if we use:

a. omnidirectional antennas
b. sectorisation with 120° antennas
c. sectorisation with 60° antennas.

Assume that only the cells from the first tier can cause co-channel interference. Based on the results, consider if it is effective to use sectorisation in the systems. Compare the trunking efficiency for the three options above.

solution:

$SIR = 1/K(3N)^{\gamma/2}$ where K is the number of interfering cells.
We require SIRdB > 18dB or SIR>63.1
Thus $1/K(3N)^2 > 63.1$

For a. K = 6.
Solving for N we get N>6.48 or N=7.
This gives 18 channels per cell. For GoS of 1% we get A=10.44erlangs per cell.

For b. K=2.
Solving for N we get N>3.74 or N=4.
We now have 31.5 channels per cell and 10 channels per sector.
A=4.46 erlangs per sector or 13.38 erlangs per cell.

For c, K=1.
Solving for N we get N>2.65 or N=3.
We now have 42 channels per cell and 7 channels per sector.
A = 2.501 erlangs/sector 15 erlangs per cell.

As we can see, it is effective to use sectorisation as we are able to increase the offered traffic intensity. From the system capacity point of view 60° sectors used in every cell gives the best result, but clearly the equipment cost would double when compared with 120° sector. Also recall the handover issue.

Erlang B Traffic Table

Maximum Offered Load Versus B and N
B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109	6.514	8.191
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230	7.856	9.800
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369	9.213	11.42
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86	13.33	16.31
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04	14.72	17.95
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22	16.11	19.60
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41	17.50	21.24
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61	18.90	22.89
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85	27.33	32.81
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06	28.74	34.46
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28	30.15	36.12
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50	31.56	37.78
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72	32.97	39.44
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94	34.39	41.10
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16	35.80	42.76
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39	37.21	44.41
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61	38.63	46.07
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84	40.05	47.74
31	14.94	16.60	17.44	19.85	21.19	22.83	25.77	29.17	32.14	35.07	41.46	49.40
32	15.63	17.34	18.21	20.68	22.05	23.73	26.75	30.24	33.28	36.30	42.88	51.06
33	16.34	18.09	18.97	21.51	22.91	24.63	27.72	31.30	34.43	37.52	44.30	52.72
34	17.04	18.84	19.74	22.34	23.77	25.53	28.70	32.37	35.58	38.75	45.72	54.38
35	17.75	19.59	20.52	23.17	24.64	26.44	29.68	33.43	36.72	39.99	47.14	56.04
36	18.47	20.35	21.30	24.01	25.51	27.34	30.66	34.50	37.87	41.22	48.56	57.70
37	19.19	21.11	22.08	24.85	26.38	28.25	31.64	35.57	39.02	42.45	49.98	59.37
38	19.91	21.87	22.86	25.69	27.25	29.17	32.62	36.64	40.17	43.68	51.40	61.03
39	20.64	22.64	23.65	26.53	28.13	30.08	33.61	37.72	41.32	44.91	52.82	62.69
40	21.37	23.41	24.44	27.38	29.01	31.00	34.60	38.79	42.48	46.15	54.24	64.35
41	22.11	24.19	25.24	28.23	29.89	31.92	35.58	39.86	43.63	47.38	55.66	66.02
42	22.85	24.97	26.04	29.09	30.77	32.84	36.57	40.94	44.78	48.62	57.08	67.68
43	23.59	25.75	26.84	29.94	31.66	33.76	37.57	42.01	45.94	49.85	58.50	69.34

44	24.33	26.53	27.64	30.80	32.54	34.68	38.56	43.09	47.09	51.09	59.92	71.01
45	25.08	27.32	28.45	31.66	33.43	35.61	39.55	44.17	48.25	52.32	61.35	72.67
46	25.83	28.11	29.26	32.52	34.32	36.53	40.55	45.24	49.40	53.56	62.77	74.33
47	26.59	28.90	30.07	33.38	35.22	37.46	41.54	46.32	50.56	54.80	64.19	76.00
48	27.34	29.70	30.88	34.25	36.11	38.39	42.54	47.40	51.71	56.03	65.61	77.66
49	28.10	30.49	31.69	35.11	37.00	39.32	43.53	48.48	52.87	57.27	67.04	79.32
50	28.87	31.29	32.51	35.98	37.90	40.26	44.53	49.56	54.03	58.51	68.46	80.99
51	29.63	32.09	33.33	36.85	38.80	41.19	45.53	50.64	55.19	59.75	69.88	82.65
52	30.40	32.90	34.15	37.72	39.70	42.12	46.53	51.73	56.35	60.99	71.31	84.32
53	31.17	33.70	34.98	38.60	40.60	43.06	47.53	52.81	57.50	62.22	72.73	85.98
54	31.94	34.51	35.80	39.47	41.51	44.00	48.54	53.89	58.66	63.46	74.15	87.65
55	32.72	35.32	36.63	40.35	42.41	44.94	49.54	54.98	59.82	64.70	75.58	89.31
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
58	35.05	37.76	39.12	42.99	45.13	47.76	52.55	58.23	63.31	68.42	79.85	94.30
59	35.84	38.58	39.96	43.87	46.04	48.70	53.56	59.32	64.47	69.66	81.27	95.97
60	36.62	39.40	40.80	44.76	46.95	49.64	54.57	60.40	65.63	70.90	82.70	97.63
61	37.41	40.22	41.63	45.64	47.86	50.59	55.57	61.49	66.79	72.14	84.12	99.30
62	38.20	41.05	42.47	46.53	48.77	51.53	56.58	62.58	67.95	73.38	85.55	101.0
63	38.99	41.87	43.31	47.42	49.69	52.48	57.59	63.66	69.11	74.63	86.97	102.6
64	39.78	42.70	44.16	48.31	50.60	53.43	58.60	64.75	70.28	75.87	88.40	104.3
65	40.58	43.52	45.00	49.20	51.52	54.38	59.61	65.84	71.44	77.11	89.82	106.0
66	41.38	44.35	45.85	50.09	52.44	55.33	60.62	66.93	72.60	78.35	91.25	107.6
67	42.17	45.18	46.69	50.98	53.35	56.28	61.63	68.02	73.77	79.59	92.67	109.3
68	42.97	46.02	47.54	51.87	54.27	57.23	62.64	69.11	74.93	80.83	94.10	111.0
69	43.77	46.85	48.39	52.77	55.19	58.18	63.65	70.20	76.09	82.08	95.52	112.6
70	44.58	47.68	49.24	53.66	56.11	59.13	64.67	71.29	77.26	83.32	96.95	114.3
71	45.38	48.52	50.09	54.56	57.03	60.08	65.68	72.38	78.42	84.56	98.37	116.0
72	46.19	49.36	50.94	55.46	57.96	61.04	66.69	73.47	79.59	85.80	99.80	117.6
73	47.00	50.20	51.80	56.35	58.88	61.99	67.71	74.56	80.75	87.05	101.2	119.3
74	47.81	51.04	52.65	57.25	59.80	62.95	68.72	75.65	81.92	88.29	102.7	120.9
75	48.62	51.88	53.51	58.15	60.73	63.90	69.74	76.74	83.08	89.53	104.1	122.6
76	49.43	52.72	54.37	59.05	61.65	64.86	70.75	77.83	84.25	90.78	105.5	124.3
77	50.24	53.56	55.23	59.96	62.58	65.81	71.77	78.93	85.41	92.02	106.9	125.9
78	51.05	54.41	56.09	60.86	63.51	66.77	72.79	80.02	86.58	93.26	108.4	127.6
79	51.87	55.25	56.95	61.76	64.43	67.73	73.80	81.11	87.74	94.51	109.8	129.3
80	52.69	56.10	57.81	62.67	65.36	68.69	74.82	82.20	88.91	95.75	111.2	130.9
81	53.51	56.95	58.67	63.57	66.29	69.65	75.84	83.30	90.08	96.99	112.6	132.6
82	54.33	57.80	59.54	64.48	67.22	70.61	76.86	84.39	91.24	98.24	114.1	134.3
83	55.15	58.65	60.40	65.39	68.15	71.57	77.87	85.48	92.41	99.48	115.5	135.9
84	55.97	59.50	61.27	66.29	69.08	72.53	78.89	86.58	93.58	100.7	116.9	137.6
85	56.79	60.35	62.14	67.20	70.02	73.49	79.91	87.67	94.74	102.0	118.3	139.3
86	57.62	61.21	63.00	68.11	70.95	74.45	80.93	88.77	95.91	103.2	119.8	140.9
87	58.44	62.06	63.87	69.02	71.88	75.42	81.95	89.86	97.08	104.5	121.2	142.6
88	59.27	62.92	64.74	69.93	72.82	76.38	82.97	90.96	98.25	105.7	122.6	144.3
89	60.10	63.77	65.61	70.84	73.75	77.34	83.99	92.05	99.41	107.0	124.0	145.9
90	60.92	64.63	66.48	71.76	74.68	78.31	85.01	93.15	100.6	108.2	125.5	147.6

[End of Question 5]