Spatial Reuse Efficiency Calculation for Multihop Wireless Networks

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***Abstract*—A multihop network provides an increase in the spatial and frequency resource reuse as compared to a single-hop network. However, a precise quantification of the benefit obtained in terms of spatial reuse is still an open issue. In this paper, a mathematical analysis is carried out in order to derive the spatial reuse efficiency of a multihop wireless network. It is demonstrated through both approximate and exact analysis, that for an unbounded network, the reuse efficiency of the wireless system increases with the number of multiple hops, M, and the spatial protection margin, Δ, defined around the receiver. Significantly, it has been found that even in case of an infinitely large spatial protection margin, the obtained reuse efficiency is a finite value and is limited by the number of multiple hops in the communicating link.**

I. Introduction

A multihop wireless network exploits the properties of multihop relaying between a source an destination, for efficient resource reuse in the wireless network. The transmission distance of the communicating pair is less in case of a multihop network as compared to an equivalent single-hop design. For a given receiver sensitivity level, this results in reduced power requirement at the transmitter; and/or an increase in the transmission data rate. The reduction in the power requirement of a communicating pair enables parallel transmission by many users, which in-turn results in an increase in the reuse of resources [1]. Apart from quantifying the increase in the resource reuse, it is also imperative to measure the potential increase in the spatial reuse of resources. There have been some attempts [2], [3] in the recent past, but due to the absence of a clear model, the spatial reuse efficiency calculation has not been tackled so far. Also, to the best knowledge of the authors, there has been no precise study that relates the increase in the spatial reuse with the number of hops in the multihop link. In this paper, a simple but a general multihop end-to-end link is considered and the spatial reuse benefit of the multihop model is calculated with respect to an equivalent single-hop network. It should be noted that parallel transmissions also lead to an increase in interference [4]. Hence, in order to reduce the effects of interference in the multihop design, a *Protocol Model* is considered in this work [5].

Section II describes the multihop wireless network in detail. The mathematical analysis and the analytical results for calculating the reuse efficiency is shown in Section III and Section IV respectively, whereas the conclusions are written in Section V.

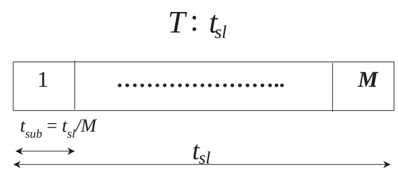

Fig. 1. TDMA frame structure for single-hop and multihop system

II. System Model

In a multihop network, the information from source node to destination node travels in multiple hops, wherein, all the remaining wireless nodes in the system act as potential relays. A node cannot transmit and receive data simultaneously, and also, it can receive data from only one of the other nodes at any instant. Hence, a time division multiple access (TDMA) based multihop wireless network is considered in the system model. In order to combat the interference coming from other node pairs, a *Protocol Model* is considered, wherein, a spatial protection margin/exclusion range ratio, denoted by Δ, is defined around each communicating receiver node. Any other potential transmitter within the exclusion range is prevented from transmitting using the same resource (same time instant in case of a TDMA system). A simple multihop scenario is shown in Fig. 2 and in Fig. 3, wherein, there are M multiple hops of equi-length, d, between the source node, P, and destination node, Q. An equivalent single-hop link between P and Q has a length, $d' = Md$. Both Fig. 2 and Fig. 3 also show a single-hop connection between P and Q along with the exclusion range circle (dark gray area in Fig. 2 and light gray area in Fig. 3) defined around the receiving node. It can also be noted from Fig. 2 and Fig. 3 that in the single-hop scenario, P and Q are the centers of the transmission and the exclusion range circles - because of space constraint, they are represented as ellipse.

III. Mathematical Analysis

Multihop networks aim to maximize the spatial reuse efficiency. The reuse factor for any time instant (TS) is determined by the exclusion range around the receiver. In the multihop scenario, a TS of duration, t_{sl}, is subdivided into M minislots, t_{sub}, i.e., $t_{\mathrm{sub}} = t_{\mathrm{sl}}/M$, wherein, each of the minislot is used for one of the M hops, as shown in Fig. 1. A transmission density parameter, δ, is defined as the ratio of

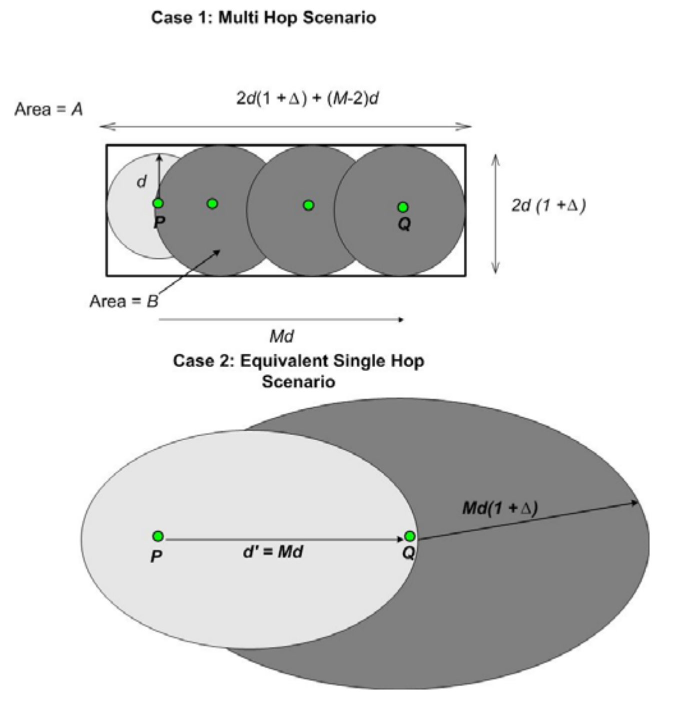

Fig. 2. Rectangular representation for single hop and multihop transmission between source, P and destination, Q

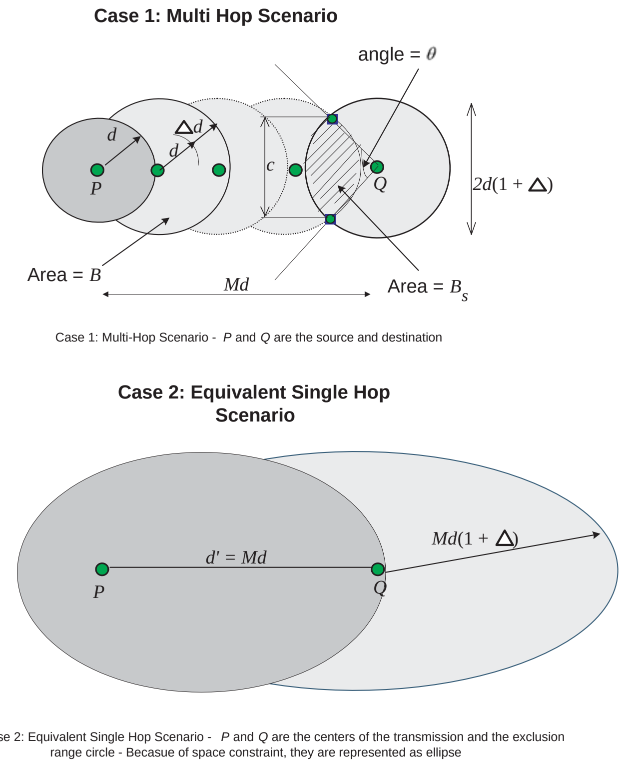

Fig. 3. Single-hop and multihop transmission between source, P and destination, Q

the time duration used for data transmission per hop, t_{sub}, to the area of the exclusion region, B, per hop. The radius, r, of the exclusion region circle is $r = d(1+\Delta)$. It follows that, $B = \pi d^2 (1+\Delta)^2$. For the multihop (mh) scenario, this results in, $\delta_{\text{mh}} = \frac{t_{\text{sub}}}{B} = \frac{t_{\text{sl}}}{M\,B}$ and for the single-hop (sh) scenario, this results in $\delta_{\text{sh}} = \frac{t_{\text{sl}}}{\pi M^2 d^2 (1+\Delta)^2} = \frac{t_{\text{sl}}}{M^2 B}$; the units for the transmission density being defined as – channel utilization in seconds/square meter. The channel transmission rate, ζ, for the multihop case is therefore defined as:

$$\zeta_{\text{mh}} = \frac{1}{\delta_{\text{mh}}\,A_{\text{mh}}} = \frac{M\,B}{t_{\text{sl}}\,A_{\text{mh}}} \tag{1}$$

where A_{mh} is the total exclusion area required to be able to serve the entire link. Determination of A_{mh} is a mathematically challenging task. Hence, in this paper, we present both an approximate and exact analysis for calculating A_{mh}. It should be however noted that the channel transmission rate for the equivalent single-hop scenario is equal to the area defined by the exclusion range. Hence, $\zeta_{\text{sh}} = \frac{1}{t_{\text{sl}}}$.

A. Approximate Analysis

In an approximate analysis, the total area 'occupied' for transmission in case of multihop network can be approximated by a rectangle [6], as shown in Fig. 2. Hence, $A_{\text{mh}} = 2d(1+\Delta)+(M-2)d)(2d(1+\Delta))$. The corresponding channel transmission rate for the multihop scenario is calculated as follows:

$$\begin{aligned}\zeta_{\text{mh}} &= \frac{M\pi d^2(1+\Delta)^2}{t_{\text{sl}}\,(2d(1+\Delta)+(M-2)d)\,(2d(1+\Delta))} && (2)\\ &= \frac{1}{t_{\text{sl}}\left((\frac{4}{M\pi}) + \frac{2(M-2)}{M\pi(1+\Delta)}\right)} && (3)\end{aligned}$$

The spatial reuse efficiency, η, is calculated from the channel transmission rate as:

$$\eta = \frac{\zeta_{\text{mh}}}{\zeta_{\text{sh}}} = \frac{1}{\frac{4}{M\pi} + \frac{2(M-2)}{M\pi(1+\Delta)}} \tag{4}$$

A calculation of the limit for an infinite number of hops results in:

$$\lim_{M\to\infty} \eta(M) = \frac{\pi(1+\Delta)}{2} \tag{5}$$

Before we analyze the results obtained from the above equations, we also derive the spatial reuse efficiency using an exact analysis.

B. Exact Analysis

Under the exact analysis, total area 'occupied' for transmission in case of multihop model is given by $A_{\text{mh}} = MB - (M-1)B_s$, where B_s indicates the overlapping area between two exclusion range circles, as shown in Fig. 3. For a multihop scenario, there is a certain amount of overlapping between two adjacent exclusion region circles of the same link which results in a reduction in the occupied area. Hence, the term $(M-1)B_s$ has been subtracted from $M\,B$ while calculating A_{mh}. Note that Fig. 3 shows a specific scenario where all the nodes are in a straight line. However, there is no difference in the result even if the multihop link form a skewed line as long as the overlapping area remains the same. Since the distance between the centers

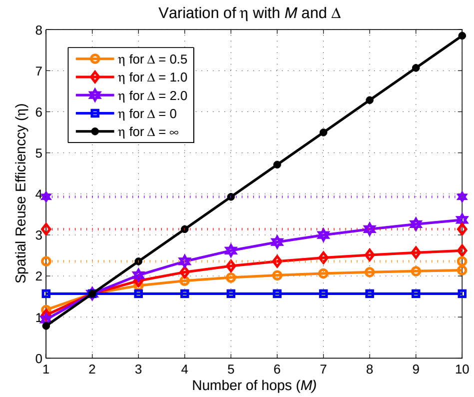

Fig. 4. Variation of spatial reuse efficiency, η under approximate analysis with spatial protection margin, Δ, and number of multiple hops per link, M

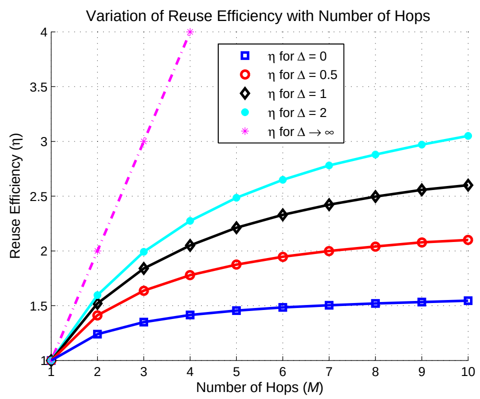

Fig. 5. Variation of spatial reuse efficiency, η under exact analysis with spatial protection margin, Δ, and number of multiple hops per link, M

of all adjacent circles are separated by the transmission distance, d, the distance between the points of intersection of the overlapping circles is $c = 2\sqrt{((1+\Delta)d)^2 - (d/2)^2}$. The angle θ (formed by the lines joining the center of a circle to the point of intersections of the two overlapping circles; as shown in Fig. 3 at the point Q) is calculated as $\theta = 2\arcsin\left(\frac{c/2}{(1+\Delta)d}\right) = 2\arcsin\left(\sqrt{1-(0.5/(1+\Delta))^2}\right)$ and is given in radians. The overlapping area is then given by $B_s = ((1+\Delta)d)^2(\theta - \sin\theta) = B\left(\frac{\theta-\sin\theta}{\pi}\right)$. For mathematical simplicity, an intermediate variable, p, is defined such that $p = \frac{\pi}{\theta-\sin\theta}$, and is used in further analysis. Hence, B_s can be written in a simplified form as, $B_s = \left(\frac{B}{p}\right)$. A direct comparison of the channel transmission rate of the single-hop and the multihop scenario yields the value of η:

$$\eta = \frac{\zeta_{\mathrm{mh}}}{\zeta_{\mathrm{sh}}} = \frac{Mp}{M(p-1)+1}; \quad M \geq 1; p \geq 1 \tag{6}$$

IV. Results

Fig. 4 plots the reuse efficiency obtained from approximate analysis for varying number of hops, as derived in eqn. (4), along with the upper bound for different Δ values, shown in eqn. (5). It can be seen that the presence of a multihop component in the system increases the reuse efficiency. Also, it can be observed from Fig. 4 that for different commonly used values of Δ ($0.5 \leq \Delta \leq 1.0$) [6], the multihop reuse gain obtained with 4 hops is nearly 90 % of that obtained with an infinite number of hops. Given the practical problems of overhead and delay in the multihop network, this is a very significant result which indicates that it is more efficient to use a multihop network with a maximum of 4 to 5 hops in a multihop wireless network. In addition, it can be observed that irrespective of the value of Δ, a two-hop system always attains a constant η of 1.57 (i.e., $\pi/2$), as compared to an equivalent single-hop network. This is an important result which explains why a two-hop architecture should always be preferred over the current single-hop design, especially in case of a cellular network.

Eqn. (6) exhibits the relationship of η with M and Δ, as per exact analysis, and is plotted in Fig. 5. It can be seen that for any value of Δ, the presence of multihop component increases the spatial reuse efficiency as compared to an equivalent single-hop network design. For an asymptotic case of $\Delta \rightarrow \infty$, $p \rightarrow 1$ for each of the multiple hops, and hence, $\eta \rightarrow M$ for these multiple hops; i.e., the maximum increase in η for the multihop system is equal to the maximum number of multiple hops between any communicating link, which is a non-intuitive but an important result.

V. Conclusions

The calculation of η from both approximate and exact analysis provide a quantitative insight into the spatial reuse gain obtained from a multihop wireless architecture. It is shown that the multihop reuse gain increases with an increase in the number of multiple hops, M, and the spatial protection margin, Δ. This is a very significant result, because larger the Δ, the higher is the expected SINR at the receiver. This means that in cases where a high SINR is required, primarily in high rate packet services, the multihop scenario clearly works in favor of achieving high spectral efficiencies. In addition, it has been shown that a two-hop wireless network (or with higher number of hops) always provides an increase in the reuse efficiency compared to a single-hop design, irrespective of the value of spatial protection margin.

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