eWARPE – Energy-efficient Weather-aware Route Planner for Electric Bicycles

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***Abstract*— Cycling, as a very attractive green form of transportation, is also one of the most sustainable. Electric bicycles, the most popular electric vehicles, subscribe to this type of transportation, being highly environmentally friendly. They have several advantages when compared to traditional bicycles, but also a weak point in terms of long battery (re)charging duration. Consequently power-saving solutions for electric bicycles are of high research interest. In this context, this paper proposes a novel energy-efficient weather–aware route planner (eWARPE) for electric bicycles. The solution makes use of the weather information in order to recommend the optimal departure time that allows the cyclist to avoid the adverse weather conditions and to maximize the energy savings of the electric bicycle. Note that the departure time is in a user-configurable time interval. The departure time can be recommended for a preferred route introduced by the user or for a route built in eWARPE based on user input. The proposed solution was validated through numerical analysis. Moreover, a survey was conducted in order to assess the impact of the adverse weather conditions on cyclists and to measure how the cyclists will benefit from the proposed solution.**

Keywords-component; electric bicycles, energy efficient, route planner, weather-aware

I. INTRODUCTION

Cycling is considered one of the most sustainable forms of transportation [1] and is beneficial both for the individual and for the environment. It can be the answer to many problems of the nowadays society such as greenhouse gas emissions, traffic congestion, parking, etc. Lately, a modern form of cycling using electric bicycles has gained important popularity. Electric bicycles are most popular in China (i.e. 92% of the world electric bicycles sales), but new research reports show that there is and there will be a worldwide increase of electric bicycles' popularity in the next years [2]. In comparison with other green vehicles, electric bicycles have lower energy cost per distance travelled [3] and avoid other additional costs (e.g. parking, insurance, registration, etc). In consequence, it is not a surprising fact that electric bicycles are the most popular among all electric vehicles [4] (EV).

Electric bicycles have many benefits. They are environmentally friendly like traditional bicycles, have no gas emissions, and provide health benefits for their users, as they encourage physical exercise. In addition, electric bicycles improve the traditional riding experience, especially for the case of people who are not so fit, in hilly terrain or in bad weather conditions (e.g. riding against the wind). However, bad weather conditions like strong wind or rainfall are not pleasant for the rider and they are identified among the main de-motivators for cycling [5].

Moreover, despite their benefits, electric bicycles have a weak point related to the same aspect that makes them capable of providing some of the already mentioned advantages to the cyclists: the battery. More precisely, battery charging periods last between 2 and 6 hours [3]. This is a relative long period and makes performing research to find power-saving solutions for electric bicycles of very high interest.

In this context, this paper proposes a novel **e**nergy-efficient **W**eather–aw**AR**e route **P**lanner solution for **E**lectric bicycles - eWARPE. The proposed solution makes use of the weather information in order to recommend the optimal departure time that allows the cyclist to avoid the adverse weather conditions (such as wind, rain fall, etc.) and to maximize the energy savings of the electric bicycle. Note that the departure time is in a user-configurable time interval. Wind in particular has a great influence on the electric bicycle power consumption. Therefore reduced wind speed and a more favorable wind direction are preferable not only to improve the cyclist experience, but also to save the bicycle battery power.

Technical solutions such as cycling route planners [6] [7] were developed with the purpose to promote cycling. They enable the cyclists to find a route between start and destination based on their preferences: more cycling facilities, fastest route, less hilly terrain, greenest route, etc. eWARPE represents a step forward for the cycling route planners, going beyond planning the route itself (how to get from point A to point B). eWARPE is planning the optimal departure time for the route: when to leave from point A to get to point B on the previously planned route in order to avoid the adverse weather conditions and to maximize the energy savings. Moreover, to the best knowledge of the authors, this is the first route planner that is targeting electric bicycles. However it can be used by all cyclists in general, energy saving being translated into cyclist effort reduction.

The proposed solution requires certain flexibility in the departure time. In leisure/fitness cycling or even in cycling for shopping purposes flexibility is usual not an issue. Even in cycling to work the flexibility in departure time in a given

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time interval should not be a problem as in the nowadays society many jobs have flexible working time. However, the flexibility in departure time was checked in a survey involving 20 subjects. The goals of the conducted survey were more extended and are detailed in Chapter IV. The survey demonstrated that cyclists are greatly affected by poor weather conditions (mainly wind and rain fall) and showed cyclists' interest in eWARPE, validating its benefits from a subjective analysis point of view. The great benefits of eWARPE in terms of numbers are shown in Chapter V.

II. Related Work

The solution proposed in this paper subscribes to the major class of vehicle routing solutions. Vehicle routing aims to find the most convenient path from start to destination based on certain criteria. Vehicle routing problem is well represented in the literature and a large plethora of solutions have been proposed. However, the energy-efficient routing for EVs is not much explored. A static energy-efficient routing solution for EVs was proposed in [8]. The road network is modeled as a weighted directed graph, where the weights are given by an energy function that takes into consideration distance and elevation. A shortest path algorithm based on this graph is then implemented.

Lately, V2X communications (i.e. vehicle to vehicle – V2V –, vehicle to infrastructure – V2I – or infrastructure to vehicle communications – I2V –) capabilities allowed for advanced dynamic and real-time routing solutions based on more accurate information regarding real-time traffic conditions and events or road characteristics and conditions. Such solutions are proposed in [9], [10] and [11]. The first two approaches are dedicated to internal combustion engine-powered vehicles and their aim is to reduce fuel consumption and gas emissions. It is expected that these solutions to be brought in the context of EVs as basically same external factors that influences fuel consumption and gas emissions also affects the energy consumption. The latter approach [11] is an energy-efficient routing solution dedicated to EVs. Machine learning techniques are employed in the computation of the most energy efficient route that integrates static map information and database information containing previously driving experiences: road conditions and characteristics, traffic conditions, charging stations. The data collection process is done via V2V/V2I communications.

As bicycles are a category of vehicles with different facilities, routing solutions have been specially designed for them in the form of cycling route planners. Some cycling route planners allow for a single-criterion routing, the more complex ones allow for multi-criteria routing [6], [7].

In [6], the authors proposed a web-based cycling route planner meant to promote cycling in Vancouver, Canada. This solution enables users to find a cycling route from start to destination based on their preferences. Users can select one of the following preferences: shortest path route, restricted maximum slope, least elevation gain, least traffic pollution and most vegetated route. Separately, users can select between two road types: cycling-friendly roads that encompasses cycling lanes and roads considered safe by the cyclists and municipal planners, and all roads. The preferences were decided based on the survey that determines main cycling motivators and de-motivators [5].

BBBike [7] is a cycle route planner originally developed for Berlin, Germany but then extended to 200 cities worldwide based on OpenStreetMap[1] data. It can be used either online or as desktop/android application and allows for a route selection based on multiple criteria: shortest route (default setting), road surface, street category (similar to road type but more than two types are considered), avoidance of unlit streets, avoidance of traffic lights and green routes preference. The latter two criteria are available for Berlin only.

None of the discussed solutions consider weather factor in reducing the energy consumption or in the latter case in improving the cycling experience, despite clear demonstration that cyclists are much affected by weather [5].

III. eWARPE Architecture

eWARPE is envisioned as a smartphone application, fact reflected in the architecture of the solution. This solution represents a step forward for the cycling route planners. eWARPE goes beyond planning the route itself, how to get from point A to point B. eWARPE is planning the optimal departure time for the route: when to leave from point A to get to point B on the previously planned route. The departure time is chosen from a user configurable time interval so that the cyclist will get the best possible cycling experience in that interval (avoid the adverse weather conditions) and will maximize the energy savings of the electric bicycle. The architecture of eWARPE (Figure 1) with the focus on this latter functionality is detailed next.

A. Proposed Architecture – Functional Blocks

User Interface is the front-end that enables the cyclist to introduce the input information represented by the route information: the path (how to get from start to destination) and the departure time interval (the time interval when cyclist is available to take the selected path). *User Interface* allows for planning the path, route itself, based on the following information: start, destination and route preferences (e.g. more cycling facilities, fastest route, etc) or for directly loading a preferred route. This basic functionality of eWARPE can be developed from scratch or ensured by integrating an open source route planner such as BBBike [7]. If the cyclist has a preferred route this can be loaded directly in the *User Interface.* The route information is sent to the *Recommendation Module* that feeds back the departure time not long before the smallest edge of the departure time interval. The departure time is announced by the *User Interface* as an alarm message. Details about the weather conditions and energy consumption for the route when leaving at the recommended departure time – (route, recommended departure time) – and (route, T_i), where T_i is in departure time interval and the difference T_i - T_{i-1} is user configurable, are displayed as well by *User Interface*. These details are

[1] http://www.openstreetmap.org

provided to the *User Interface* by the *Recommendation Module.*

Network Interface is the component responsible with retrieving real-time traffic information and route-aware weather information. The information about the route is provided by the *Recommendation Module* when it requires the aforementioned information from the *Network Information.* The route-aware weather information is the weather forecasted by the closest meteorological station to the route. If the route is closed to more than one meteorological station then the route is split and the weather information is retrieved from more meteorological stations. This increases the accuracy of the solution.

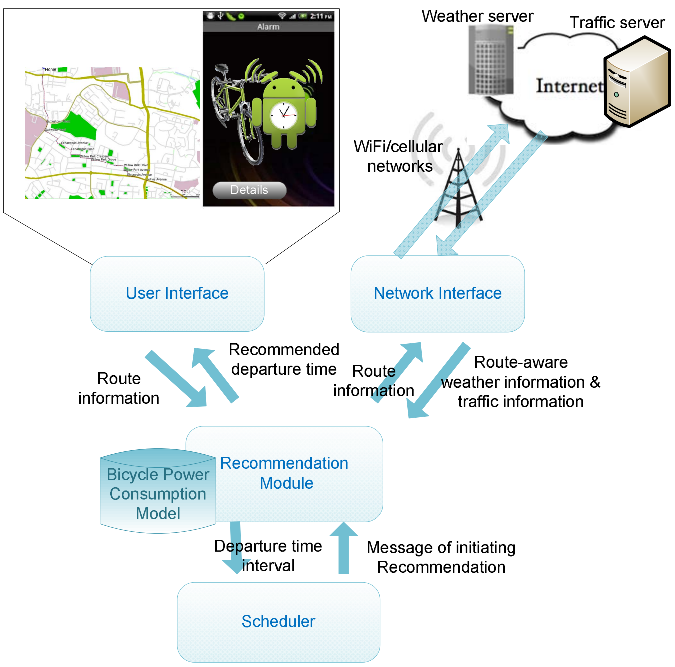

Figure 1. eWARPE Architecture

Recommendation Module is the component that, based on the information it is provided with, determines the departure time (in the given time interval) so that the cyclist will be provided with the best weather conditions possible in terms of cycling experience and energy-efficiency. While computing the optimum departure time, the *Recommendation Module* computes also the energy consumption of (route, T_i). This information is needed for the more detailed report.

The energy consumption of the route is determined based on the *Bicycle Power Consumption Model* detailed in section II.B and travel time that is possibly influenced by the traffic volume. This is the reason why information about real-time traffic is retrieved. However, this subject is open as the influence of traffic volume is not the same on the bicycles as on the cars. In addition, where cycling lanes available traffic volume might not influence at all.

The computation of optimum departure time is controlled by a *Scheduler*. The *Scheduler* checks the departure time interval and ensures that *Recommendation Module* is requiring the information from *Network Interface* not long before the smallest edge of this interval. As such, the accuracy of the information received from the *Network Interface* will increase and consequently the accuracy of the computation will increase, too. The *Scheduler* timings can be configured by the users via the *User Interface.* The interaction between the functional blocks is illustrated for a better understanding in Figure 2.

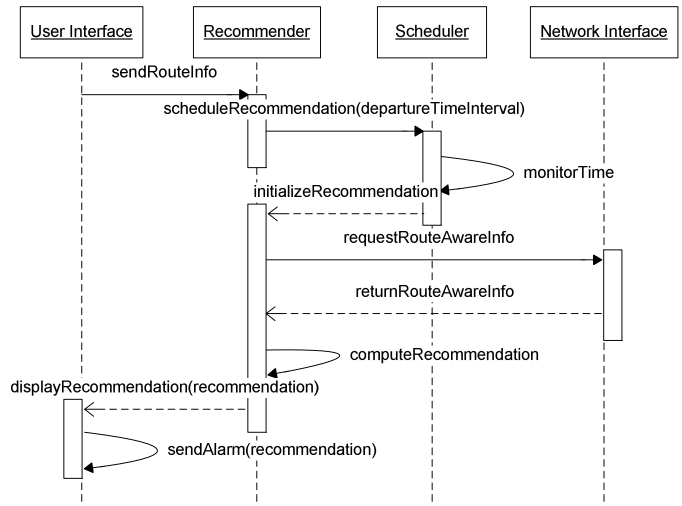

Figure 2. Interaction Between Functional Blocks

B. Bicycle Power Consumption Model

In this paper we used as baseline the same power consumption that was employed in computing the theoretical power consumption of an electric bicycle in [3] and [12]. According to this model, the total power consumption: P_{total} – eq. (1) is the sum of three powers: the power needed to overcome the air drag: P_{drag} – eq. (2), the power needed to overcome the slope: P_{hill} – eq. (3) and the power needed to overcome the surface resistance: $P_{fricition}$ – eq. (4). The drawback of this model is that for simplification purposes, it was assumed the scenario of riding into headwind. So the eq. (2) is not reflecting accurately the wind influence on power consumption. We enhanced this baseline model by considering the wind direction. Thus, the power needed to overcome the air drag is computed in our consumption model using: $P_{drag\text{-}enhanced}$ – eq. (5), as taken from [13].

$$P_{total} = P_{drag} + P_{hill} + P_{fricition} \quad (1)$$

$$P_{drag} = \left[0.5 \cdot C_d \cdot D \cdot A \cdot (v_g + v_w)^2\right] \cdot v_g \quad (2)$$

$$P_{hill} = (g \cdot G \cdot m) \cdot v_g \quad (3)$$

$$P_{fricition} = (g \cdot m \cdot R_c) \cdot v_g \quad (4)$$

Thus, P_{drag} is replaced by $P_{drag\text{-}enhanced}$ described in eq. (5).

$$P_{drag\text{-}enhanced} = \left[0.5 \cdot C_d \cdot D \cdot A \cdot (v_g + v_w \cdot \cos(D_w - D_B))^2\right] \cdot v_g \quad (5)$$

TABLE I. POWER CONSUMPTION MODEL NOTATIONS

Notation	Explanation
C_d	Drag coefficient

D	Air density (kg/m^3)
A	Frontal Area (m^2)
v_g	Ground velocity (m/s)
v_w	Wind speed (m/s)
g	Gravitational acceleration = 9.81 (m/s^2)
G	Slope grade
m	The overall weight: cyclist + bicycle + additional equipments (kg)
R_c	Rolling coefficient
D_w	Direction of wind (degrees)
D_B	Direction of bicycle (degrees)

IV. SURVEY

Following the guidelines in [14], a survey study was conducted with the main goals listed below:

- to demonstrate the existence of the problem: poor weather conditions affect cyclists;
- to determine the dimensions of the problem (measure the impact of the poor weather conditions on the cyclists) and consequently to measure the need for the proposed solution;
- to validate the cyclists' interest in the proposed solution that aims at guiding them in addressing the aforementioned problem;
- to measure the benefits of the proposed solution: do cyclists think that the proposed solution will improve their cycling experience?

A number of 20 subjects (Male=11, Female=9) between 19 and 34 years old were interviewed so far. Subjects that presented no interest in cycling were not considered. For the purpose of this study, only subjects who are active cyclists or did cycle in the past and would still consider cycling in the future were considered. The subjects were asked to select the class they pertain to from the following classes [5]: regular cyclist (52 or more one-way trips per year), frequent cyclist (12-51 one-way trips per year), occasional cyclist (1-11 one-way trips per year) and potential cyclist (never in the past year). The distribution per classes of the subjects was: 35% regular cyclists, 15% frequent, 30% occasional and 20% potential. All subjects live in Dublin, Ireland where the climate is known to be mild, with no extreme temperatures[2]. This makes cycling possible throughout the year, but the problems for cycling are the frequent rain falls and strong winds.

A. The problem and its dimensions

During the interview, the subjects were asked what they consider to be the main disadvantages of cycling. The answers in the order of their popularity were:

- negative impact of the poor weather, rain and wind mainly (identified by 50% of the subjects);
- safety issues (45% of the subjects);
- cycling is uncomfortable (10% of the subjects);
- reasons of maintenance (10% of the subjects);
- "not suitable for longer trips" (5% of the subjects);

The top answers to the question what are the main factors that negatively affect them when cycling were:

- poor weather conditions (55% of the subjects mentioned this negative factor);
- driver behavior (25%);
- not enough cycling facilities ("not enough space to cycle", "roads not suitable for cycling") (25%) ;
- lack of safe places for locking the bicycles (10%).

Note that before subjects answered these two aforementioned questions, no thought about weather conditions was induced. The questions were free with no suggested answers.

The negative effect of poor weather conditions on cycling was expressed by the subjects as follows: "more difficult to cycle", "causes health problems", "make cycling more dangerous", "uncomfortable", "not suitable for wearing fancy clothes", "carried things get wet". When they were asked to measure this effect, the subjects ranked in average with 4 the level of disturbance caused by the adverse weather conditions on a scale from 1 to 5 (not disturbed at all – very disturbed).

B. The interest in eWARPE and its benefits

The results presented above demonstrate the need for the proposed solution as the weather is an important issue for cycling. Moreover, only 15% of the subjects declared that they are usually tightly constrained by time so their departure time is not flexible, the rest of 85% said that their trips by bicycle are flexible so the departure time is flexible given a time interval. However, all the subjects stated that they would choose the departure time that corresponds to the best weather conditions (least chances of precipitation falls and less influence of the wind) in a certain time interval.

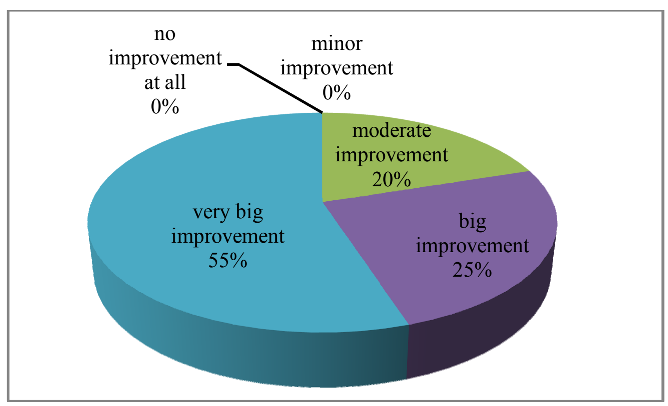

Figure 3. Proposed solution improvement on subjects' cycling experience

Thus the interest showed by the subjects in the proposed solution is not surprising: 19 out of 20 subjects stated that they are interested in such solution and that this solution would determine them to cycle more or to be more inclined to cycle. Only one subject said that he would be somehow interested in the solution, but this solution would not determine him to cycle more. In addition, the participants to the interview considered that the proposed solution would improve their

[2] http://www.met.ie/

cycling experience. On a scale of 1 to 5 (no improvement at all to very big improvement) the improvement was rated as follows: 5 – 25%, 4 – 55%; 3 – 20% (as illustrated in Figure 3), the average rate being 4.1 (big improvement).

V. CASE STUDY

In order to validate the benefit brought by the eWARPE in terms of energy savings, we consider a case-study performed on the route of one of the first participants to the survey that is cycling daily, Monday to Friday, from home to Dublin City University (DCU) and back. The aim was to show in terms of energy saving the benefits of using the solution proposed in the paper. As most of the participants in the survey, the subject declared that the departure time is flexible, in a certain interval of time, and that will definitely choose to go when there are fewer chances to rain and when the influence of the wind is less. The subject stated that the usual leaving time from home to DCU is in the interval [7:30 – 9:00] and from DCU to home is in the interval [17:30 – 19:00]. The cyclist also stated that traffic volume does not affect the time of the trip because on the segments of road with increased traffic there are cycle lanes.

The weather data was taken from Dublin City Airport meteorological station (EIDW) as this is the closest station to the subject's route. The weather data from January 2012 till June 2013 (18 months) was retrieved[3]. Saturdays, Sundays, public holidays and DCU closing days were eliminated from the weather data.

Figure 4. Case-study route

The route was drawn in BBBike@Dublin (Figure 4), the BBBike web-based cycle route planner for Dublin, and it was confirmed by the cyclist. In order to validate the benefits in terms of energy saving brought by eWARPE, the total energy consumption determined by air drag/route is computed. Note that only the power consumption needed to overcome the air drag is considered as this is where the wind influence is reflected. Moreover, the additive relationship between the powers (eq. (1)) in the power consumption model allows for this type of analysis: if the $P_{drag\text{-}enhanced}$ decreases/increases with a value the P_{total} decreases/increases with the same value. The route is segmented and the energy consumption is computed for each segment of the road and then cumulated in order to obtain the total energy consumption. The energy consumption/segment of road was computed based on the power consumption needed to overcome the drag (eq. (5)) and the time needed to travel the segment of road. The following parameter values were used in the computation:

- D = 1.247 kg/m^3, A = 0.4 m^2, C_d = 1 (typical values [3]).
- v_g= 5m/s – the speed was set to this value for all the segments of the road as it was considered the speed in average
- D_B is computed using the great circle technique [15], based on the GPS coordinates retrieved from BBBike@Dublin
- v_w, D_w are taken from the weather data
- the time in which a segment of road is travelled on was given by the cyclist and validated also in BBBike@Dublin

The results of the case-study are summarized in TABLE II. The least energy consumption/month is taken as reference point in computing both *min* energy saving/month and *max* energy saving/month. The least energy consumption/month is computed by cumulating the least energy consumption values obtained for each day of the month. Whereas the least energy consumption in a day is obtained by choosing the optimum departure time from the aforementioned time intervals so that to have the lowest possible wind influence during cycling. The max energy saving/month is computed as the difference between the worst energy consumption/month and the least energy consumption/month. The worst energy consumption/month is obtained as the cumulative of each day of the month when the cyclist is leaving at the most inappropriate time so that the wind influence is worst possible as per the time intervals specified. Min energy savings are obtained as the difference between the next least energy consumption obtained and the least energy consumption/month. For a better understanding of the energy saving obtained, both min and max values are presented in terms of full charges of an electric bicycle (4th column of TABLE II. and Figure 5). The usual capacity of an electric bicycle is considered to be around 300Wh[4].

TABLE II. CASE-STUDY RESULTS

Month	Min energy saving (Wh)	Max energy saving (Wh)	Benefit in terms of full charges ≈ 300Wh
Jan-12	529.59	1173.34	[1.77 –3.91]
Feb-12	193.64	681.13	[0.65 – 2.27]
Mar-12	253.21	740.31	[0.84 – 2.47]
Apr-12	216.42	829.81	[0.72 – 2.77]
May-12	280.61	930.46	[0.94 – 3.1]
Jun-12	240.71	900.97	[0.8 – 3]
Jul-12	236.64	805.69	[0.79 – 2.69]
Aug-12	239.73	953.22	[0.8 – 3.18]

[3] METAR/Synop Weather Information for EIDW, http://weather.gladstonefamily.net/site/EIDW

[4] http://www.electricbike.com/watt-hours/

Sep-12	298.37	1047.52	[1 – 3.5]
Oct-12	214.48	718.93	[0.71 – 2.4]
Nov-12	366.85	943.02	[1.22 – 3.15]
Dec-12	208.59	574.9	[0.7 – 1.91]
Jan-13	294.39	953.74	[0.98 – 3.18]
Feb-13	196.02	736.86	[0.65 – 2.46]
Mar-13	261.33	836.73	[0.87 – 2.79]
Apr-13	383.42	1452.6	[1.28 – 4.84]
May-13	503.66	1507.78	[1.68 – 5.03]
Jun-13	186.86	911.37	[0.62 – 3.03]

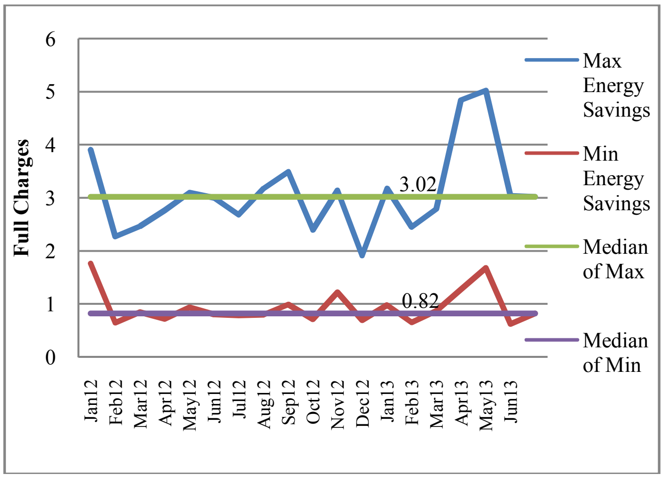

Figure 5. Monthly possible energy savings in terms of full battery charges for January 2012 – June 2013

On average, in the period of time analyzed (January 2012 – June 2013), monthly energy savings between 1 to 3 full battery charges are possible to be obtained through a solution such as the one proposed in the paper. Moreover, in the time period analyzed, there were only six days were no variations in the direction and the speed of wind in the time interval [7:30 – 9:00] (four days in 2012, two days in 2013), while in the time interval [17:30 – 19:00] there was only a single day recorded (19^{th} of April 2012) with no wind variations. Moreover, as we have seen significant energy savings are possible if the departure time is selected in such a way that the wind influence is minimal. This validates the solution proposed.

VI. CONCLUSIONS

This paper proposes a novel energy-efficient weather–aware route planner for electric bicycles – eWARPE. eWARPE represents a step forward for the cycling route planners, going beyond planning the route itself (how to get from point A to point B). eWARPE is planning the optimal departure time for the route: when to leave from point A towards point B on the previously planned route. The solution makes use of the weather information in order to recommend the optimal departure time that allows the cyclist to avoid the adverse weather conditions and to maximize the energy savings of the electric bicycle. Note that the departure time is in a user-configurable time interval. A survey validated the proposed solution from a subjective perspective, all the interviewed subjects showing interest in the solution. In addition, they considered that eWARPE would bring a big improvement on their cycling experience. A case-study performed based on the input received from one of the interviewed subjects proved the great benefits of eWARPE in terms of energy-efficiency.

VII. ACKNOWLEDGEMENT

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