Rotationally Invariant 3D Shape Contexts Using Asymmetry Patterns

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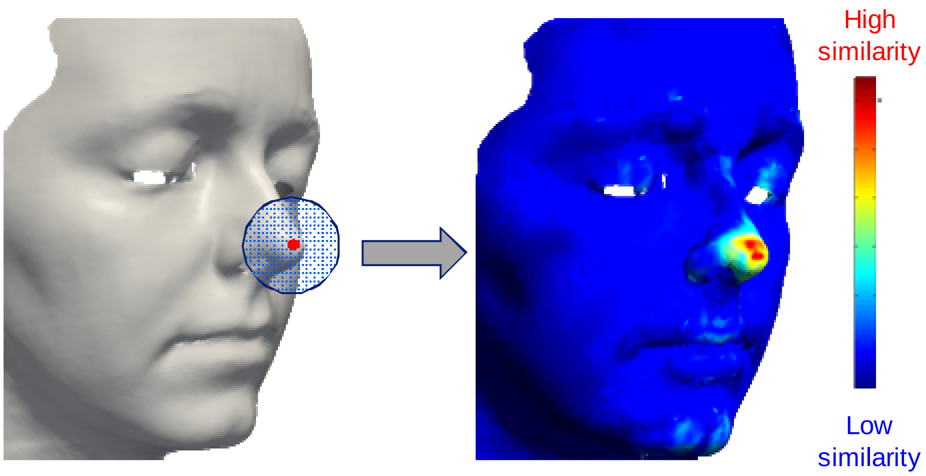
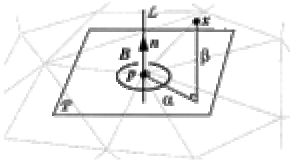
3D Geometric Descriptors

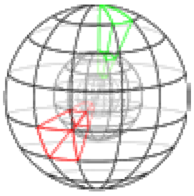


- We look for local properties that allow distinguishing a given point from the rest of points of the mesh

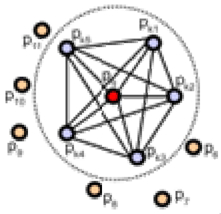
3D Geometric Descriptors

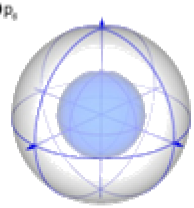


- Distance-based
  - Spin Images (SI)
    A. Johnson et al. IEEE T Pattern Anal 21 (1999) 433–449
  - 3D Shape Contexts (3DSC)
    A. Frome et al. In: Proc. ECCV (2004) 224–237
  - Unique Shape Contexts (USC)
    F. Tombari et al. In: Proc. 3DOR (2010) 57–62



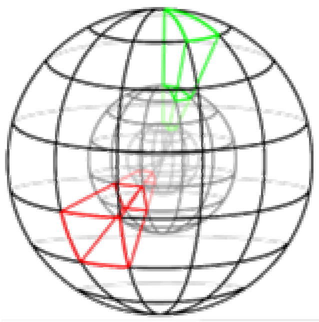
- Orientations-based
  - Point Feature Histograms (PFH)
    R. Rusu et al. In: Proc. IROS (2008) 3384–3391
  - Fast Point Feature Histograms (FPFH)
    R. Rusu et al. In: Proc. ICRA (2009) 3212–3217
  - Signature of Histograms of Orientations (SHOT)
    F. Tombari et al. In: Proc. ECCV (2010) 356–369





F.M. Sukno, J.L. Waddington and P.F. Whelan. **Comparing 3D Descriptors for Local Search of Craniofacial Landmarks**. ISVC 2012, pp 92-103.

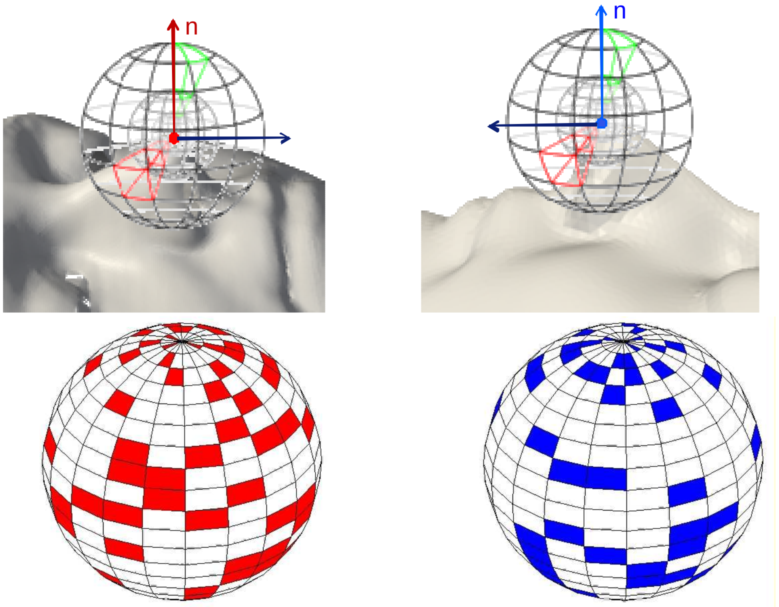
3D Shape Contexts (3DSC)



- Histogram of distances on a spherical support region
- Logarithmic spacing of radial bins
  - Bins closer to the interest point are smaller to capture detail
  - More distant bins are bigger, to capture “context”
- Volumetric normalization
  - To compensate for sampling density
  - To compensate for bin size

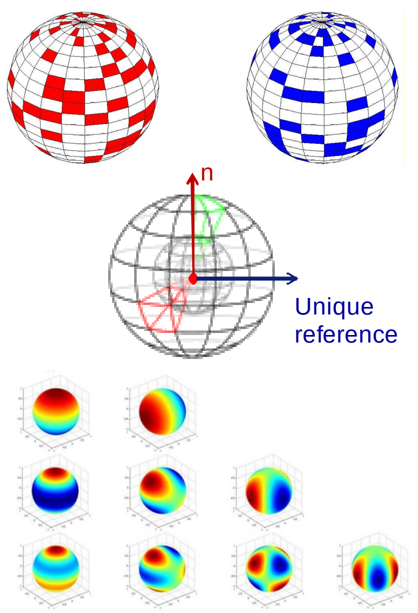
A. Frome, D, Huber, R. Kolluri, et al. **Crecognizing objetcs in range data using regional point descriptors**. ECCV 2004, pp 224-237

3DSC Orientation and Matching



3DSC Azimuth ambiguity

- 3DSC (Frome et al. 2004)
  - Test all possible rotations and pick the best match
- Unique Shape Contexts (Tombari et al. 2010)
  - Resolve the azimuth origin by SVD + sign disambiguation
  - Lower accuracy than 3DSC
- Harmonic Shape Contexts (Frome et al. 2004)
  - Use spherical harmonics and keep just the module
  - High computational cost



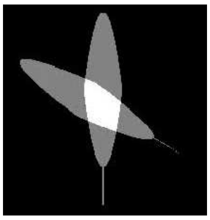
Asymmetry Patterns Shape Contexts (APSC)

- We define a new family of descriptors by extracting asymmetry patterns from 3DSC
- APSC properties:
  - Rotationally invariant by construction
    - Faster comparison (matching)
  - Small increase in computational cost w.r.t. 3DSC (typically 5% to 10%)
  - Can be defined from a variety of patterns
    - Useful when targeting different types of points
- We can get several APSC descriptors for the price of one (with a small overhead).
- Experiments on 3D facial landmark localization

Measuring symmetry

- We start from a measure of approximate rotational symmetry in 2D, presented by Guo et al (2010):

$$S_c(\mathbf{m}, \phi) = \frac{\mathrm{Area}(\mathbf{m} \cap R(\mathbf{m}, \phi))}{\mathrm{Area}(\mathbf{m})}$$



- $S_c(\mathbf{m},\phi)$: continuous degree of rotational symmetry of shape **m** at a rotation ϕ
- $R(\mathbf{m},\phi)$: Rotation of shape **m** by an angle ϕ

Q. Guo, F. Guo, and J. Shao. **Irregular shape symmetry analysis: Theory and application to quantitative galaxy classification**. IEEE TPAMI 2010, 32(10):1730–1743.

Measuring symmetry

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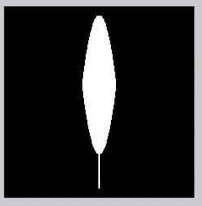
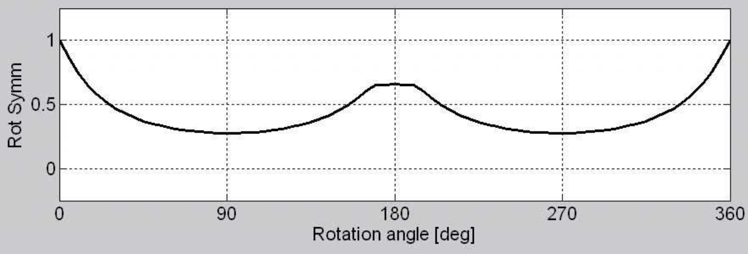
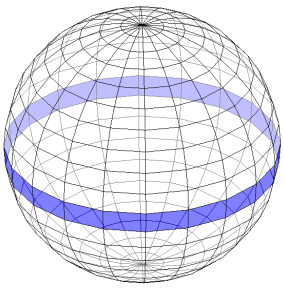


No need to set the origin for the rotation !



Adapting symmetry to 3DSC

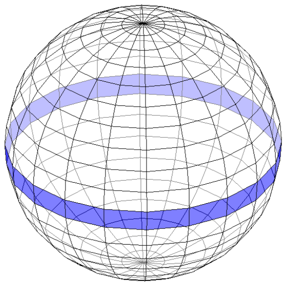


- Consider the bins at fix elevation and radius
  - We get a stripe that varies with Azimuth
  - The bin values are a sequence **m** of N_A values (the number of Azimuth bins)

$$m_j = x_{i,j,k}, \quad m_j \geq 0 \forall j \in [1;N_A]$$

  - $x_{i,j,k}$ are the bins of a 3DSC
    - i = Elevation
    - j = Azimuth
    - k = Radius

Adapting symmetry to 3DSC



- We can redefine symmetry as:

$$\mathcal{S}(\mathbf{m},a) = \frac{\sum_j \min(m_j, m_{j+a})}{\sum_j m_j}$$

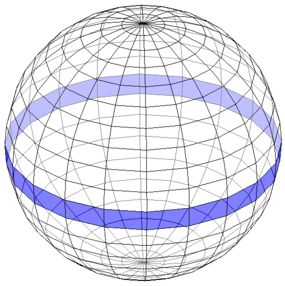
  - Rotation is replaced by a discrete shift by “a” units
  - Additions j+a are operations modulo N_A
- The formula for its complement (asymmetry) becomes:

$$\mathcal{A}(\mathbf{m},a) = 1 - \mathcal{S}(\mathbf{m},a) = \frac{1}{2}\frac{\sum_j |m_j - m_{j+a}|}{\sum_j m_j}$$

- We can drop the ring-constant elements to get:

$$\mathcal{A}_1(\mathbf{m},a) = \sum_j |m_j - m_{j+a}|$$

Asymmetry patterns

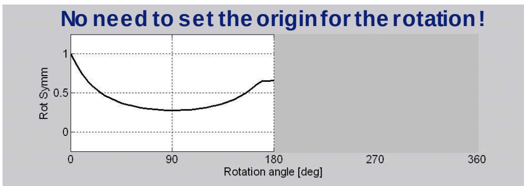


- The difference of absolute values of a ring and a shifted version of itself measures its asymmetry at that shift

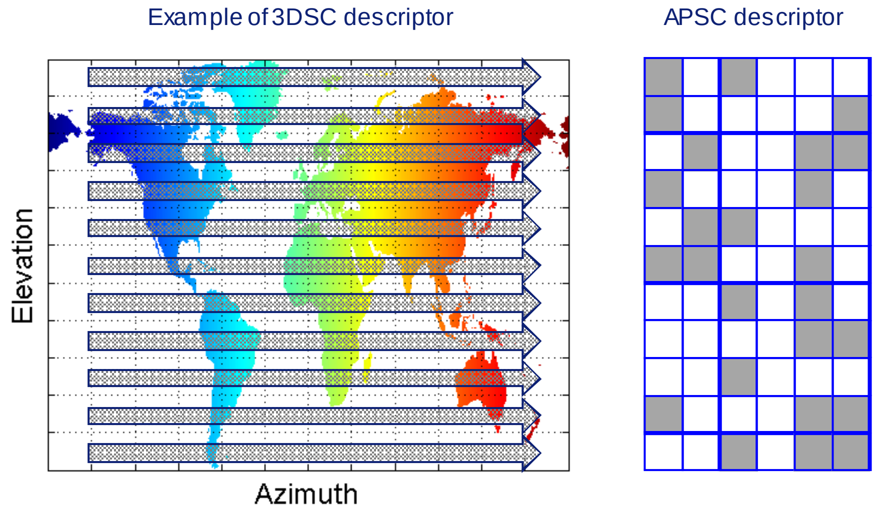
$$\mathcal{A}_1(\mathbf{m},a) = \sum_j |m_j - m_{j+a}|$$

- We can define an asymmetry pattern as follows:

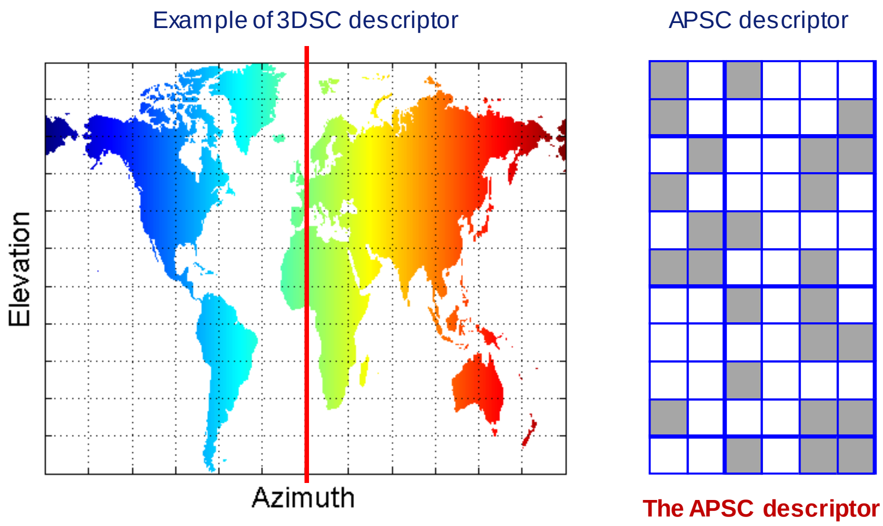
$$\mathcal{P}_A(\mathbf{m}) = \mathcal{A}_1(\mathbf{m},1), \mathcal{A}_1(\mathbf{m},2), \ldots, \mathcal{A}_1(\mathbf{m}, \lfloor \frac{N_A}{2} \rfloor)$$



World-Map example

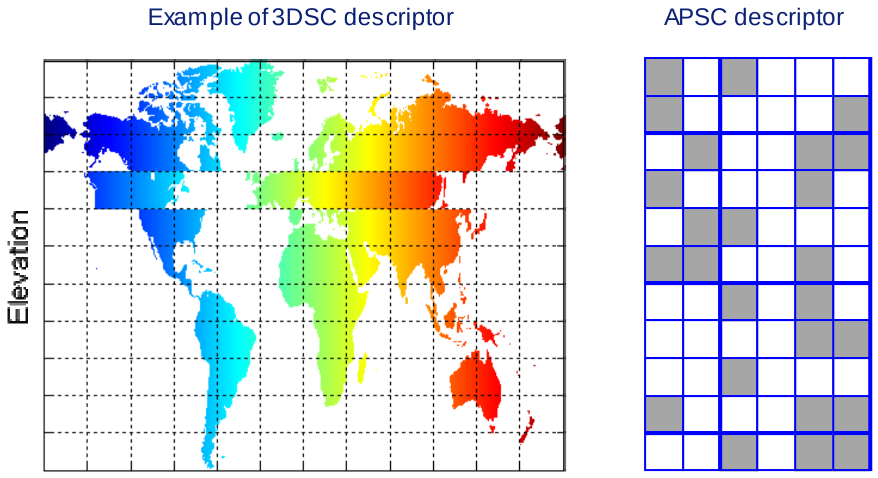


Side effects of rotational invariance



The APSC descriptor remains unchanged

Side effects of rotational invariance



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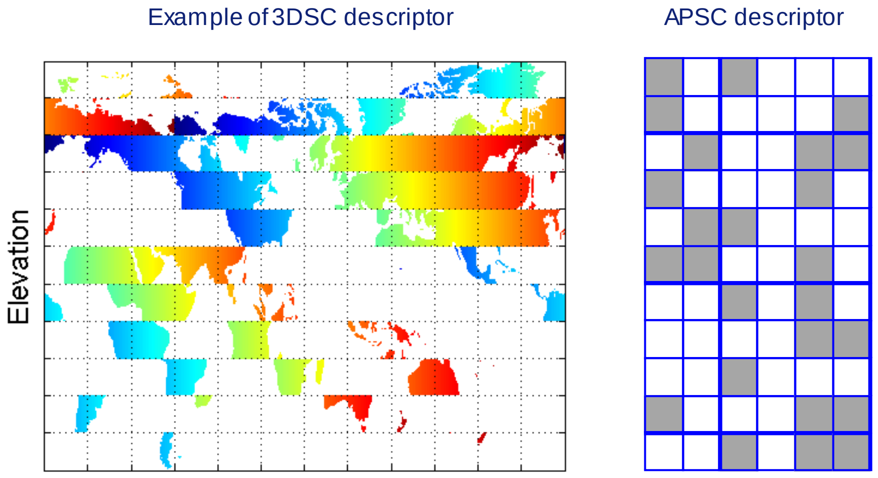
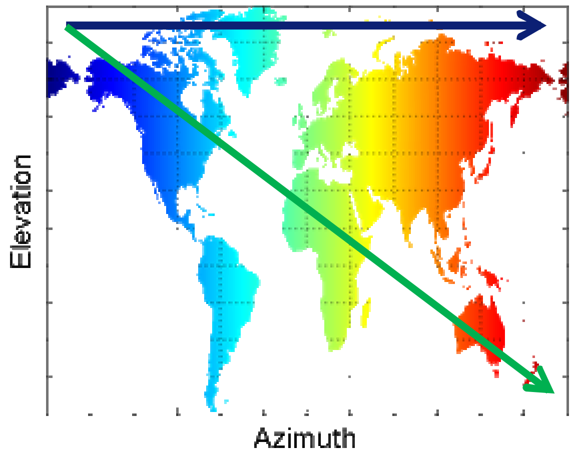


The APSC descriptor remains unchanged

Selecting the spatial patterns (1-ring)

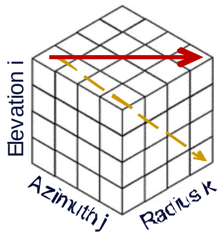
Example of 3DSC descriptor ($x_{i,j,k}$)



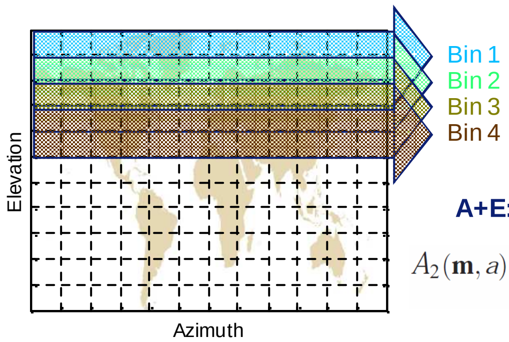
The disconnection is not resolved but the spatial relations captured are different

Spatial Pattern:

A: $m_j = x_{i,j,k}$



2-ring patterns



Spatial Pattern:

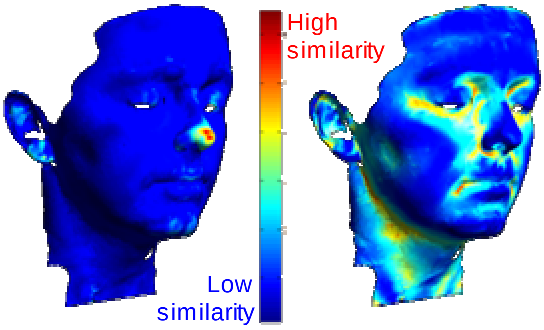
A+E: $m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+1,j,k}$

$$A_2(\mathbf{m}, a) = \sum_j |m_{1,j} - m_{1,j+a}| + |m_{2,j} - m_{2,j+a}|$$

Abbreviation	Sequence(s) equation	Description
D_{AR}	$m_j = x_{i,j,k+j}$	Azimuth-Radius diagonal
D_{AER}	$m_j = x_{i+j,j,k+j}$	Azimuth-Elevation-Radius diagonal
A+E	$m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+1,j,k}$	Azimuth ring + Elevation neighbors
A+R	$m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i,j,k+1}$	Azimuth ring + Radial neighbors
A+D_{AER}	$m_{1,j} = x_{i,j,k}, \quad m_{2,j} = x_{i+j,j,k+j}$	Azimuth ring + Azim-Elev-Rad diagonal

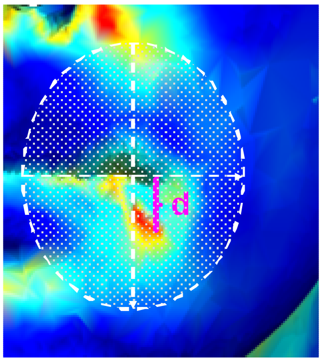
Evaluation of local accuracy

Example of similarity maps computed using an <u>average descriptor</u>



Nose tip Mouth corners

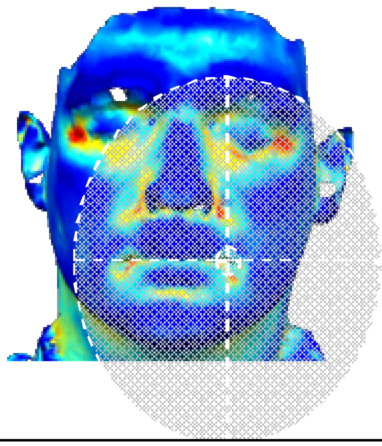
Similarity maps are typically not as good as the one for the nose tip

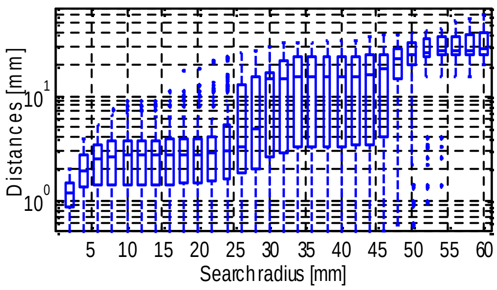


We restrict the analysis to a local neighborhood of the target

Definition: Expected Local Accuracy

- Is the expected distance from the vertex obtaining the maximum score to the ground truth position, but only searching on a neighbourhood of radius r





Expected Local Accuracy

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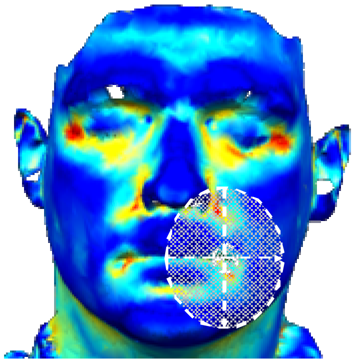
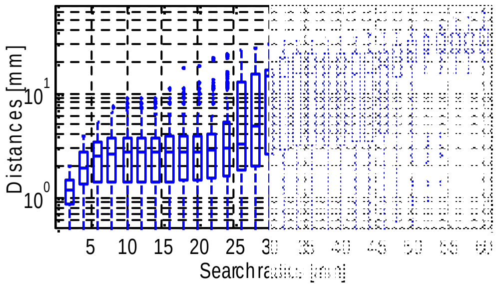
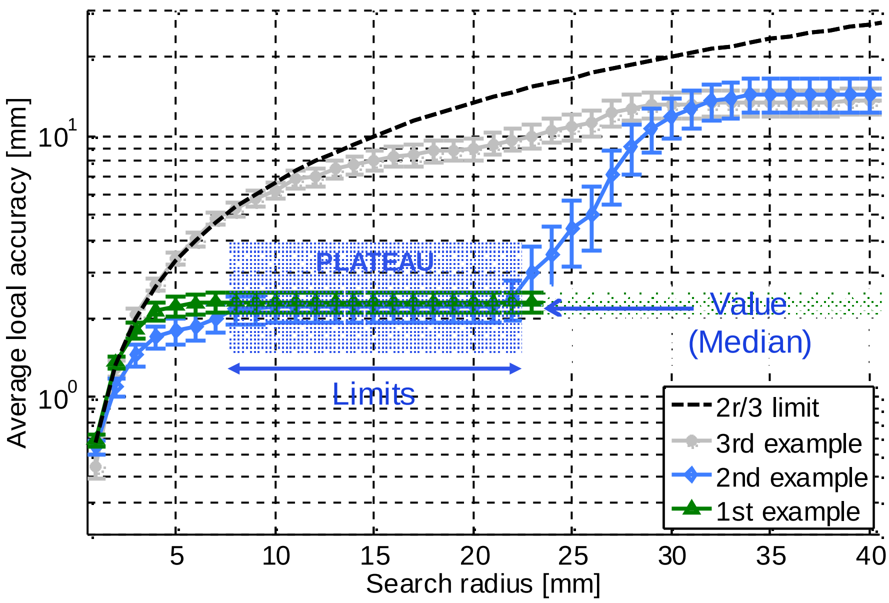
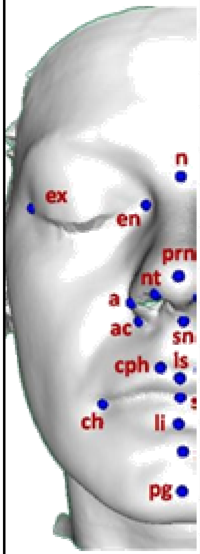




Expected local accuracy curves

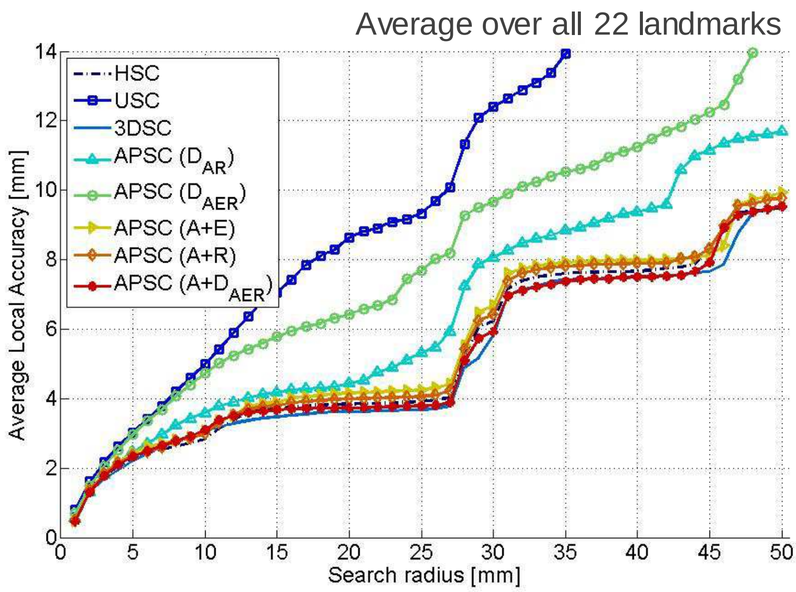


Results (144 facial scans, 6-fold cross validation)



Lmk	HSC	USC	3DSC	APSC				
				D_{AR}	D_{AER}	A+E	A+R	A+D_{AER}
en (2)	**1.3*** **(2 - 24)**	1.9 (3 - 25)	1.4 (3 - 25)	1.5 (3 - 25)	1.5 (3 - 25)	**1.4*** **(3 - 24)**	**1.3** **(3 - 25)**	**1.3*** **(3 - 25)**
ex (2)	4.5 (16 - 90)	n.p	4.3 (13 - 88)	2.9 (6 - 67)	3.9 (19 - 48)	5.4 (13 - 88)	4.7 (14 - 89)	**3.1*** **(8 - 88)**
n	1.8 (3 - 200)	4.6 (5 - 12)	**1.5** **(3 - 200)**	**1.6*** **(4 - 200)**	**1.6*** **(4 - 64)**	2.3 (4 - 200)	2.0 (3 - 200)	**1.7*** **(4 - 200)**
a (2)	**1.4*** **(3 - 26)**	n.p	1.4 (4 - 27)	2.9 (6 - 12)	n.p	2.1 (4 - 25)	1.8 (4 - 26)	2.0 (6 - 26)
ac (2)	**2.1*** **(5 - 25)**	5.8 (14 - 25)	4.7 (9 - 25)	9.0 (16 - 24)	n.p	2.3 (7 - 25)	2.1 (4 - 11)	5.1 (14 - 25)
nt (2)	**2.0** **(4 - 8)**	12.2 (14 - 200)	8.0 (14 - 200)	6.9 (12 - 200)	7.5 (11 - 200)	2.3 (5 - 8)	2.2 (5 - 9)	6.6 (11 - 200)
prn	1.4 (3 - 200)	1.4 (2 - 200)	**1.2** **(2 - 200)**	1.3 (3 - 200)	**1.3*** **(2 - 200)**	**1.3*** **(2 - 200)**	**1.3*** **(2 - 200)**	1.3 (3 - 200)
sn	1.8 (4 - 200)	n.p	**1.6** **(4 - 55)**	**1.8*** **(4 - 22)**	2.0 (5 - 16)	1.9 (3 - 200)	1.9 (3 - 200)	1.9 (4 - 200)
ch (2)	3.8 (11 - 22)	2.4 (4 - 42)	**2.1** **(5 - 19)**	2.5 (9 - 29)	2.9 [illegible]	2.8 (6 - 18)	2.9 [illegible]	**2.3*** **(5 - 28)**
cph (2)	**2.1** **(4 - 9)**	13.3 (20 - 34)	8.4 (18 - 200)	7.1 (17 - 86)	7.0 (16 - 59)	n.p	7.7 (16 - 200)	2.7 (5 - 8)
li	5.0 (16 - 51)	2.7 (7 - 48)	2.3 (5 - 10)	4.4 (16 - 37)	3.4 (11 - 45)	4.9 (10 - 15)	4.8 (9 - 15)	3.8 (15 - 95)
ls	4.1 (6 - 14)	n.p	**2.3*** **(8 - 46)**	2.7 (8 - 15)	**2.2** **(6 - 11)**	5.2 (14 - 200)	5.7 (10 - 54)	3.8 (7 - 200)
sto	**2.7*** **(6 - 14)**	2.9 (8 - 46)	**2.2** **(8 - 78)**	**2.5*** **(7 - 17)**	6.1 (14 - 40)	4.0 (9 - 14)	4.5 (11 - 89)	3.1 (12 - 54)
sl	5.4 (10 - 54)	**3.0** **(10 - 18)**	3.2* (11 - 27)	5.5 (13 - 79)	7.4 (16 - 29)	4.7 (11 - 77)	6.0 (12 - 84)	6.2 (17 - 62)
pg	7.0 (10 - 200)	11.6 (19 - 120)	**5.4** **(10 - 200)**	7.9 (19 - 200)	7.1 (15 - 200)	7.6 (15 - 26)	**5.6*** **(13 - 23)**	**5.7*** **(10 - 200)**

Global performance



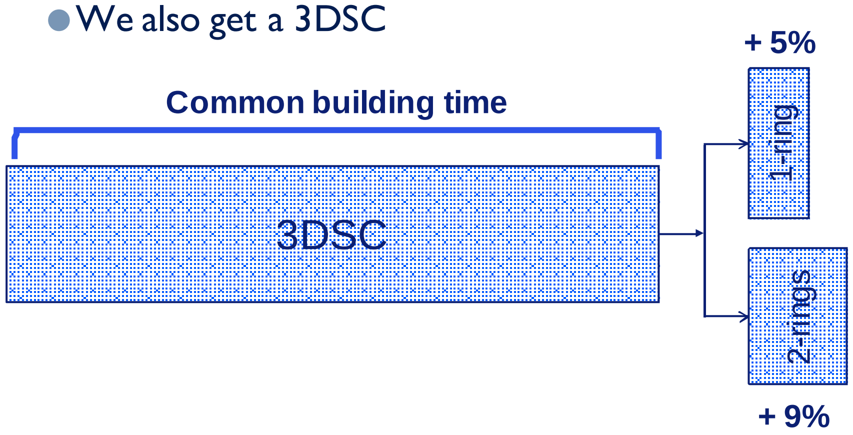
Computational complexity

- Tests on triangulated meshes averaging 21K vertices and 1.5mm mesh resolution
- Intel Xeon E5320 @1.86 GHz
- Time values normalized (3DSC time = 1.0)

Descriptor	Computation	Matching	
HSC	11.1	$\frac{N_{SH}(N_{SH}+1)}{2N_E N_A^2}$	≈0.083
USC	1.23	$\frac{1}{N_A}$	≈0.083
APSC (D_{AR})	1.05	$\frac{1}{2N_A}$	≈0.042
APSC (D_{AER})			
APSC (A+E)	1.09	$\frac{1}{2N_A}$	≈0.042
APSC (A+R)			
APSC (A+D_{AER})			

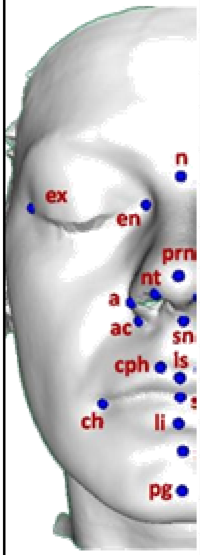
APSC computation

- Once an APSC descriptor is built
  - Additional APSC can be built at little extra cost
  - We also get a 3DSC



Results (2)

Can be computed in 1.23 times the 3DSC



Lmk	HSC	USC	3DSC	APSC				
				D_{AR}	~~D_{AER}~~	~~A+E~~	A+R	A+D_{AER}
en (2)	**1.3*** **(2 - 24)**	1.9 (3 - 25)	1.4 (3 - 25)	1.5 (3 - 25)	[illegible]	[illegible]	**1.3** **(3 - 25)**	**1.3*** **(3 - 25)**
ex (2)	4.5 (16 - 90)	n.p	4.3 (13 - 88)	2.9 (6 - 67)	3.9 (19 - 48)	5.4 (13 - 88)	4.7 (14 - 89)	**3.1*** **(8 - 88)**
n	1.8 (3 - 200)	4.6 (5 - 12)	**1.5** **(3 - 200)**	**1.6*** **(4 - 200)**	1.6* (4 - 64)	2.3 (4 - 200)	2.0 (3 - 200)	**1.7*** **(4 - 200)**
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prn	1.4 (3 - 200)	1.4 (2 - 200)	**1.2** **(2 - 200)**	1.3 (3 - 200)	1.3* [illegible]	1.3* [illegible]	**1.3*** **(2 - 200)**	1.3 (3 - 200)
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ls	4.1 (6 - 14)	n.p	**2.3*** **(8 - 46)**	2.7 (8 - 13)	2.2 (6 - 11)	5.2 (14 - 200)	5.7 (10 - 54)	3.8 (7 - 200)
sto	**2.7*** **(6 - 14)**	2.9 (8 - 46)	**2.2** **(8 - 78)**	**2.5*** **(7 - 17)**	6.1 (14 - 40)	4.0 (9 - 14)	4.5 (11 - 89)	3.1 (12 - 54)
sl	5.4 (10 - 54)	**3.0** **(10 - 18)**	3.2* (11 - 27)	5.5 (13 - 79)	7.4 (16 - 29)	4.7 (11 - 77)	6.0 (12 - 84)	6.2 (17 - 62)
pg	7.0 (10 - 200)	11.6 (19 - 120)	**5.4** **(10 - 200)**	7.9 (19 - 200)	[illegible]	[illegible]	**5.6*** **(13 - 23)**	**5.7*** **(10 - 200)**

Conclusions

- Individual APSC descriptors:
  - Rotationally invariant, globally comparable to 3DSC
  - Are 5% - 10% more expensive to build than 3DSC
  - Are much faster to evaluate than 3DSC and require half its memory
- Groups of APSC
  - Can be computed incrementally with little additional cost for each additional descriptor
    - The extra cost is only to build the descriptor, not to match it
  - We also get the 3DSC descriptor, in case there are no APSC with sufficient accuracy
  - Under similar performance APSC are much faster to evaluate

Craniofacial dysmorphology

- Craniofacial geometry has been suggested as an index of early brain dysmorphogenesis in neuropsychiatric disorders
  - Down syndrome
  - Autism
  - Schizophrenia
  - Bipolar disorder
  - Fetal alcohol syndrome
  - Velocardiofacial syndrome
  - Cornelia de Large syndrome
  - Brandet-Biedl syndrom
  - …

The Face3D project

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MARIE CURIE ACTIONS

Face3D RESEARCH CONSORTIUM

THANK YOU FOR YOUR ATTENTION

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