# Development of a vision system for the flexible packing of random shapes. 

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#### Abstract

In this paper we put forward a framework for the flexible packing of planar shapes, of random shape and size, under visual control. The basic aim of this system is not only to be able to produce an efficient packing strategy, but a strategy that is also flexible enough for industrial use, and as such the method taken emphasises the systems approach to dealing with industrial vision problems. This framework consists of two major components, namely a morphological based geometric packing approach used in conjunction with a heuristic packing procedure. Some of the considerations included at a heuristic level include shape ordering and shape orientation, both of which must be carried out prior to the application of the shapes to the geometric packer. The heuristic component also deals with the context information specific to our application. We will also discuss the various issues that arise from this approach, such as the systems properties and performance, within the background of some sample applications. The ideas outlined in this paper are currently been used in the development of a visually controlled intelligent packing work cell.


## 1. INTRODUCTION

The ability to manipulate objects under visual control is one of the key tasks in the successful implementation of robotic, automated assembly and adaptive material handling systems. According to Gorog [7] the strongest growth rate forecast for the use of machine vision in industry is in the area of material handling. In general $25 \%$ of typical products are too large or complex to be handled in automated part feeders and as such they must be supplied on pallets during the manufacturing process. Therefore in order to automate this part of the manufacturing process we need to develop a flexible packing strategy using machine vision.

The aim of this paper is to discuss the development of a system that will allow the automated packing of planar shapes, of arbitrary shape and size, under visual control. The ideas outlined in this paper are currently used in the development of an adaptive packing work cell which will allow the automated packing, nesting and simple assembly of such arbitrary shapes. Since the simpler packing problems, such as the pallet packing problem [5], have been shown to be NP-complete [8] we aim to produce an efficient rather than an optimal packing strategy. The main emphasis of this work is to produce a system that will be flexible enough for industrial use, and as such we emphasis the systems approach to the packing problem.

The packing system developed consists of two major components, firstly a morphological based geometric packer which takes an arbitrary shape of a given orientation and packs the shape in that orientation, this has been discussed elsewhere [15]. The second major component of the packing system consists of a heuristic packer, this component is concerned with the ordering and alignment of shapes prior to applying them to the geometric packer. This component also deals with other general considerations, such as the conflict in system constraints and the measurement of packing performance, as well as dealing with practical constraints such as the effects of the robot gripper on the packing strategy and the isotropic constraints such as
grain and pattern matching. It is the heuristic component of the packing system that we will concentrate on in this paper and this will be discussed within the context of some sample packing applications.

This paper is organized as follows, Section 2 discusses the packing strategy in terms of a systems approach to the problem. Section 3 details the heuristic component of the packing strategy, and in particular deals with the application of this procedure to the automated packing of 2-dimensional blob shapes and simple polygons into arbitrary scenes ${ }^{1}$. In Section 4 we introduce the performance measures necessary to evaluate the systems packing ability. Section 5 discusses the performance of the strategy in the context of a number of different sample applications. Finally, in Section 6 we discuss the issues arising from this approach and the conclusions we can draw from the development of such a flexible packing system.

## 2. SYSTEMS APPROACH TO FLEXIBLE PACKING

One of the key research areas in industrial vision is the development of flexible vision systems. Some of the main elements necessary to progress the development of such systems include the need for vision systems that are more adaptable and can cope with a wider range of variable products and that can work in an less constrained environment. Other key elements in these systems include the ability to manipulate arbitrary shapes and the ability to cope with unanticipated failures. In the development of such flexible systems, we should be seeking engineering solutions to standard industrial problems that are consistent with the limited capacities of our current generation of industrial vision systems [6], while at the same time developing new techniques, which are often generic, to push the bounds of vision applications.

One way to maximize the potential of the current generation of vision systems, that is when faced with a specific application demand, is to view the vision problem within a systems engineering framework. In such a framework we should try to maximise the use of contextual information available to us from the product and the manufacturing process and environment, and by doing so we hope to reduce the complexity of the application. For example, we may find that by mechanically restricting the orientation and the order of objects passed to the packing system we can greatly simplify the problem. In fact, if we could impose these constraints in a real application the packing procedure would mainly consist of the geometric component, with little requirement for a complex heuristic component in the strategy. These system demands are not always unreasonable and should always be investigated.

It is in the context of this framework that the industrial vision packing strategy has been developed. The two main components, the morphological based geometric packer and the heuristic packing component, generate a flexible technique that allows the packing of arbitrary 2 -dimensional shapes. While the technique will pack any shape presented to it, its efficiency is very much dependent on the application. Therefore we need to avail of the context information for a given application to ensure that we get an efficient packing strategy for that application, see Diagram 1.

By using the heuristic component in the packing strategy, we ensure that we get an efficient, but not necessarily an optimal solution. The main problem with using this component is the tendency to produce a set of overly complex heuristics that produce paradoxes within the system and will in turn bring the system to its knees. It is therefore necessary to keep all the logic decisions within the heuristic component as simple as possible to avoid these paradoxes. Another issue we must consider is whether we can manipulate shapes for packing and assembly without explicitly describing these shapes ?

The packing system was developed using the PROLOG+ system developed at the University of Wales [1]. This system gives a great deal of flexibility to the developer while in turn minimizing the development overhead. This allows the developer to concentrate on the problem without becoming unnecessarily entangled in the implementation details.

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Diagram 1. General packing strategy

## 3. HEURISTIC COMPONENT PACKING TECHNIQUES

As mentioned earlier, the flexible packing system consists of two main components, a morphological based geometric packing stage [15] which takes a shape of a given orientation and packs it into the scene under investigation in that orientation. The heuristic component determines the orientation and order in which the shapes are applied to the geometric packer, this we refer to as the heuristic packer. In the following discussion we aim to show how the heuristic packing component works in terms of two shape classes, those of blobs and simple polygons. It is necessary to view both these general shape classes since no secret recipe exists for all shape classes and while the geometric packing level is independent of the shape class and hence the application context, the heuristic component is much more context dependent.

### 3.1 Blob packing techniques.

This section outlines the heuristics necessary to deal with 2-dimensional binary images of random shape and size, prior to the application of the geometric packer. The approach outlined was designed specifically for an off-line packing process but the technique could equally be applied to an on-line packing process.

Since the system is off-line, all the shapes to be packed are presented simultaneously. The shapes are then classified in terms of the convex hull's bay size. The bay areas are ranked in order of size, the shape with the largest bay is the first to be applied to the geometric packer. Once the shape ordering has been decided it is necessary to orientate each shape such that a efficient local packing strategy can be implemented. Four orientation rules are used to align the shape to be packed in the scene under investigation and these are outlined below:

- Rule 1:If the current shape has no bay and if the circularity measurement of the shape is less than 1 then the shape is classed as a disk and as such no realignment need take place.
- Rule 2:If the size of the shapes largest bay is classified as significant and if the circularity value of the shapes largest bay is greater than 2, then the shape is found to have an elongated bay so it is orientated such that it lies at the same angle as the least moment of inertia of the original scene we wish to pack.
- Rule 3: If the size of the shapes largest bay is classified as insignificant, the shape is orientated such that it lies at
the same angle as the least moment of inertia of the original scene we wish to pack.
- Rule 4:If the size of the shapes largest bay is classified as significant and if the circularity value of the shapes largest bay is less than or equal to 2 , then the shape is rotated such that its largest bay lies in the same orientation as the original scenes least moment of inertia. The largest bay is always directed in towards the section of the scene with the maximum amount of unpacked space. This last requirement ensures that the shapes can be efficiently nested at a local level during the packing procedure.

Once a shape has been aligned and ordered, it is then applied to the geometric packer. Figure 1 shows an example of this approach $^{2}$ as it was applied to the packing of some basic tools into a rectangular tray. Figure 2 shows the packing of these tools into a random blob region.


Packing Density ${ }^{3}$ : 43
Shapes presented: 5
Shapes packed: 5
Performance Index: . 43
Space Usage: . 26
Figure 1. Packing tools in a rectangular tray.


Packing Density: . 395
Shapes presented: 5
Shapes packed: 5
Performance Index: . 395
Space Usage: . 194
Figure 2. Packing tools in an irregular region.

### 3.2 Polygon packing techniques.

A different packing procedure is used to pack simple polygons, that is polygons which do not contain significant bay areas and as such the approach outlined in Section 3.1 is not as efficient, although it will pack the polygon shapes. As before this procedure was designed to work with an off-line packing system but could also be applied to on-line packing applications. Unlike the previous procedure this approach also has the ability to classify the local efficiency of each shape packed and will reorientate the shape to ensure an efficient local packing configuration (this local efficiency check could also be applied to the blob packing strategy outlined in Section 3.1). In this particular application example we are packing non-uniform box shapes (squares and rectangles) into a square scene (Figure 3). Once all the shapes are presented to the packing system they are ordered according to their size, with the largest shape the first to be packed. The shapes must then be orientated into position prior to the application of the geometric packer.

In the initial versions of this packing procedure the shapes were aligned such that the least moment of inertia was matched to

[^1]the least moment of inertia of the scene under investigation, but this method proved unreliable for packing squares. This was because of the quantization effects (a $256 \times 256$ CCD camera was used) of digitizing squares produces a digital square image with a jagged edge and this can cause errors in the calculation of the moment of inertia. It was decided to align the angle of the largest face of the shape to be packed with the angle of the largest face of the scene in order to avoid the problem just discussed. The face angles for the shape, and the scene, were found by generating the hough transform of the edge image of the polygon shape under investigation. The hough transform was used due to its tolerance to local variations in straight line edge images. Once the peaks in the hough space were enhanced and separated from the background the largest peak in the hough space was found [2]. This peak corresponds to the largest face of the polygon under investigation. Since the position of the peak in hough space gives us the radial and the angular position of the largest face angle, we can therefore align the shapes largest face angle such that it lies in the same orientation as the largest face angle of the original scene we wish to pack.

Once a polygon shape was packed a local packing efficiency check was carried out. This consisted of ensuring that the number of unpacked regions within the scene was kept to a minimum. The polygon was rotated through a number of predefined rotations, and after each rotation the number of unpacked regions in the scene was checked. If a single unpacked region existed then this was considered a local optima and the local packing efficiency routine was exited and we proceed to pack the next shape. Otherwise we continue the local packing efficiency check until we are sure that when the shape is packed a minimum number of unpacked regions in the scene exist. This reduces the chance of large voids appearing in the packed scene, therefore improving its overall efficiency.

Depending on the application, we may require a more restricted subset of the approaches outlined above or the application may require a 'compound heuristic' [3] that classes the input shapes as blobs or polygons and handles the packing of those shapes accordingly.


Packing Density: . 92
Shapes presented: 6
Shapes packed: 6
Performance Index: . $\mathbf{9 2}$
Space Usage: . 806
Figure 3. Packing of non-uniform boxes in a square tray.

## 4. PERFORMANCE MEASURES

To have confidence in the global efficiency of any packing strategy, we must have some way of measuring the systems performance. Traditionally, packing performance has been measured by a number, i.e. a packing density value [13]. This is the ratio of the total area of all the packed shapes to that of the total area of the scene under investigation. This is referred to
as the worst case analysis packing measure. But a number of other performance measures have been developed in the field of operational research (see [5] for a good review of packing procedures used in operational research) to deal with classifying different heuristics when applied to the efficient packing of rectangular bins by odd sized box shapes. These fall into two main categories [10]. The first of these categories is referred to as probabilistic analysis and in this approach one assumes a density function for the problem data and establishes probabilistic properties of a heuristic such as the expected performance of the heuristic or a bound on the probability that the heuristic finds a solution within a prespecified percentage of the optimum solution. The final approach is statistical, one usually applies the heuristic on a large number of sample problems to draw some statistical inferences on the approach.

While the latter two measurements can be quite useful in a well constrained packing problem, they are of little use in dealing with the packing of arbitrary shapes. The performance measures used in our strategy are based on the traditional worst case analysis. This is now explained in detail below.

### 4.1 Packing density based performance measures.

After a packing strategy has been applied to a given scene it is classified by five values:

- Packing density
- Number of shapes presented to the scene
- Number of shapes packed in the scene
- Performance index
- Space usage

The packing density is a ratio of the optimal packing area, which is defined as the total area of all the shapes packed, to the area of the convex hull of all the shapes packed in the scene. This measure will have a maximum value of 1 . The performance index is a weighted value of the packing density. The weighting used is referred to as the count ratio, which is defined as the ratio of the total number of shapes packed to the number of shapes initially presented to the scene. This also has a maximum value of 1 . The weighting of the packing density by the count ratio allows us to account for any shapes that remain unpacked at the end of the packing session. The space usage value is a ratio of the area of the shapes packed to the total area of the scene under investigation. Again, this value has a maximum of 1.

## 5. EXPERIMENTAL RESULTS

In this section we discuss the performance of the packing strategy adopted in the context of a number of sample applications. Each application is accompanied by its corresponding performance measures, as outlined in Section 4.1. The first application is the packing of some standard household items, such as scissors, keys and pens, into a rectangular tray (Figure 4) and into an irregular scene (Figure 5).

Figures 6 to 8 show the automated nesting of simple polygon shapes using the approach outlined in Section 3.2. Figure 6 shows two possible resultant packing configurations on the application of the packing strategy to the automated nesting of a simple block jigsaw. Figure 7 shows the application of the polygon packing procedure to the assembly of a 2-dimensional 'apictorial' [16] Chinese tangram puzzle. Whereas other researchers [9] have developed specific routines to solve this puzzle, the solution shown here is based on the application of the unmodified polygon packing procedure to the puzzle. Better results could be obtained by the addition of further application specific heuristics to the packing procedure.


Packing Density: . 34
Shapes presented: 6
Shapes packed: 5
Performance Index: . 28
Space Usage: . 25
Figure 4. Packing household items in an rectangular tray.


Packing Density: . 41
Shapes presented: 6
Shapes packed: 5
Performance Index: . $\mathbf{3 4}$
Space Usage: . 248
Figure 5. Packing household items in an irregular region.

### 5.1 Pallet packing.

In Figure 8, the general purpose polygon packing procedure is applied to the extensively studied operational research problem of pallet packing [4]. The problem consists of presenting the system with a number of empty pallets which have to be packed. These pallets compromise the scene in our packing problem, in this particular example we are attempting to pack three pallets. A number of non-uniform boxes are then presented to the packing system, whose task it is to maximise the number of boxes to be packed, while at the same time ensuring the efficient packing of each pallet in the scene. Again the unmodified polygon packing procedure of Section 3.2 was applied to the problem, to produce the result shown in Figure 8. The only system constraint imposed was that all the pallets must be aligned in the same orientation prior to packing.

### 5.2 Robot gripper considerations.

One of the practical considerations that must be investigated in automated packing is the ability for the packing strategy to be robust enough to cope with a range of different types of material handling systems. The above applications assumed the use of suction pads to lift and place the objects. In general automated material handling systems make use of robotic grippers, and this adds the extra complication of allowing the robot gripper access to objects in a partially packed scene. Therefore, our strategy must make allowances for the gripper when it is in its worse case position. This occurs when the gripper is fully open after placing the object to be packed in position. This problem can be dealt with by overlaying a gripper template on the shape to be packed prior to the application of this shape to the geometric packer. The gripping template is based on the grippers open and closed position and accounts for the grippers profile as well as its worse case position.

Figure 9, show the resultant packing configuration on packing tools into a rectangular tray, when one considers the gripper profile. Although the general blob packing strategy outlined in Section 3.1 remains the same, the systems performance is weakened when we allow access room for the robot gripper. For example, compare the results of Figures 1 and 9. But this strategy does have the advantage of allowing the shapes to be unpacked from the scene in any order. A possible modification to this approach would be to pack each shape with the gripper template, but prior to packing the next shape remove the gripper template from the scene. This has the effect of improving the efficiency of the previous approach, but introduces an extra constraint in that we must unpack the shapes in the reverse order to which they were packed.


Packing Density: . 78
Shapes presented: 5
Shapes packed: 4
Performance Index: . 62
Space Usage: . 51


Packing Density: . 82
Shapes presented: 5
Shapes packed: 4
Performance Index: . 65
Space Usage: . 60

Figure 6. Automated nesting of simple polygons in a rectangular scene.


Packing Density: . 78
Shapes presented: 7
Shapes packed: 5
Performance Index: . 56
Space Usage: . 507


Packing Density: 79
Shapes presented: 7
Shapes packed: 5
Performance Index: . 56
Space Usage: . 65

Figure 7. Partial assembly of Chinese tangram puzzle.


Packing Density: . 69
Shapes presented: 6
Shapes packed: 5
Performance Index: . 57
Space Usage: . 56


Packing Density: . 41
Shapes presented: 5 Shapes packed: 4
Performance Index: . $\mathbf{3 3}$ Space Usage: . 29

Figure 9. Tool packing with worse case gripper template.

Figure 8. Pallet packing.

## 6. DISCUSSION AND CONCLUSION

This paper outlines a strategy that will allow the flexible packing of 2-dimensional arbitrary shapes. This strategy is discussed in terms of a systems approach to industrial vision. The system is still under development, although the results presented show the power of this approach when presented with a wide range of problems. The approach taken highlights the importance of maximizing the use of application specific information to produce an efficient packing strategy. The paper also outlines a technique that will allow a performance measure to be taken for each of the packing procedures implemented. This is necessary due to the fact that the heuristic approach taken will not guarantee an optimal result, therefore some measurement of performance must be taken to allow validation of the approach taken.

One of the problems with this approach is that we are trying to produce an efficient global packing strategy based on local optimization. There is the danger that the system may get trapped in a suboptimal solution. The concept of non-local assembly, which is similar in nature to the 'intelligent groping' mentioned by Penrose [11] when discussing the tiling problem, suggests that it may be occasionally necessary to view a shape that is not a local optima for the current packing stage to ensure that no errors occur in the packing process and that an efficient global packing strategy is implemented. In fact some researchers [12] have made the observation that the more complex the shape, the more likely a random strategy, such as the one outlined by Uhry [14], is to succeed. Although the development of this system is not complete, it is our belief that the combination of the packing strategy outlined in this paper in conjunction with maximizing the use of the application domain information will produce a good global packing strategy.

## 7. ACKNOWLEDGEMENTS

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[^0]:    ${ }^{1}$ The term 'scene' is used to indicate the 2-dimensional region in to which we pack the arbitrary shapes.

[^1]:    ${ }^{2}$ The spacing between packed images in a scene can be controlled to a certain extent during the geometric packing stage [15]. All packed images are spaced out so that they can be viewed more clearly.
    ${ }^{3}$ See Section 4.1 for an explanation of the performance measures.

